ME 201/MTH 281ME400/CHE400
Examples of Bessel Expansions

1 Introduction

In this notebook, we construct and plot the partial sums of a Fourier-Bessel series for two specific solutions.

We use the notebook entitled Convergence of Bessel Expansions. In this notebook we use only the functions $J_0$.

\[ \frac{d}{dr} \left( r \frac{d\psi}{dr} \right) + \lambda \psi = 0 , \quad 0 < r < a , \]

We have $\psi_n(r) = J_0(\alpha_n r / a)$, where $\alpha_n$ is the $n$th root of $J_0$. These eigenfunctions are orthogonal or orthonormal in these eigenfunctions. The coefficients in the expansion are calculated using orthogonality. The relation for the expansion is

\[ f(r) = \sum_{n=1}^{\infty} C_n \psi_n(r) , \quad \text{with} \]

2 Expansion of $f(r) = a^2 - r^2$

We now illustrate this theory by expanding the function

\[ \text{in}[1] := f[r_] := a^2 - r^2; \text{fstring} = "a^2 - r^2" ; \]

We choose the value 3 for $a$:

\[ \text{in}[2] := a = 3; \]

We find the first 51 zeros of $J_0$ and assign them to the list named $\alpha$.

\[ \text{in}[3] := \alpha = N[\text{Table}[\text{BesselJZero}[0, i], \{i, 1, 51\}]] \]


We define the eigenfunctions for Mathematica.

\[ \text{in}[4] := \psi[r_, n_] := \text{BesselJ}[0, r \times \alpha[[n]] / a] \]

We call the nth expansion coefficient coeff[[n]]. The theoretical expression is given above in equation Mathematica function NIntegrate. We also will obtain the coefficients from the analytical expression.

\[ \text{in}[5] := \text{num}[n_] := \text{NIntegrate}[f[r] \times \psi[r, n] \times r, \{r, 0, a\}] \]