Problem #1 – 10pts: Looking at Appendix.A Eq. A.15, for \( \{x\} = [x_1 \ x_2 \ldots \ x_n]^T \) and \( \varphi = \frac{1}{2} \)
\( \{x\}^T [A] \{x\} \) with \([A]\) being independent of \(x_i\)

a. Show the following for \([A]\) being an arbitrary \(n \times n\) matrix

\[
\left[ \frac{\partial \varphi}{\partial x} \right] = \left[ \frac{\partial \varphi}{\partial x_1} \ \frac{\partial \varphi}{\partial x_2} \ \ldots \ \frac{\partial \varphi}{\partial x_n} \right]^T = \frac{1}{2} \left( \{A\} + [A]^T \right) \{x\}
\]

b. Show if \([A]\) is an arbitrary and symmetric \(n \times n\) matrix

\[
\left[ \frac{\partial \varphi}{\partial x} \right] = [A] \{x\} \quad \text{and} \quad \frac{\partial^2 \varphi}{\partial x_i \partial x_j} = A_{ij} = A_{ji}
\]

Problem #2 – 5pts:

8.2-1 Let \([E']\) be 3 by 3, as for a plane stress problem. Show that Eq. 8.2-10 yields
\( [E'] = [E] \) if the material is isotropic.

\[
[E] = [T_e]^T [E'] [T_e] \quad (8.2-10)
\]

Problem #3 – 5pts:

1.3-2 Strain \( \varepsilon_x \) is given by the expression \( \varepsilon_x = \frac{\partial u}{\partial x} \). What expression for \( \varepsilon_x \) is obtained when \( u \) in a four-node plane element is given by the right-hand side of Eq. 1.3-6?

For a mesh of such elements, what can you say about interelement continuity of \( \varepsilon_x \)?

Problem #4 – 10pts: Exact solution of some equation has produced \( z = \sin(x^2+y^2) \). Assume we want to approximate \( z \) field for \( x,y \in [0.5,1] \) using a square 2D element with 1 DOF per node. Find \( a_i \) coefficients for the fittings and plot both the fitted surface and original surface in the same 3D plot.