DOE module: Practice problems

Part 1.

1) People with dysphagia can have difficulty swallowing liquids with too low a viscosity. To avoid this, "thickening agents" can be added to drinks to increase their viscosity. Assume that you have been assigned to study the effect of three factors on the thickness (viscosity) of tea. You have two possible thickening agents ("A" and "B"), two possible concentrations (1 or 2 grams in a 4 oz. drink), and two possible mixing procedures (stir and shake).

a) Which of these factors are discrete and which are continuous?

b) Design a full factorial experiment to measure the effect of these three factors on viscosity. In your design include columns to aid in calculating the two-factor and three-factor interaction effects.

c) How many degrees of freedom are in your design?

2) The effects of the amount of curing agent and the curing time on the adhesion strength of dental brackets were studied using a full factorial experiment. The experiment and its results are summarized in the tables below.

Factor	**Level**	
	-1	+1
X1. amount of curing agent (mg)	50	100
X2. curing time (seconds)	15	60

TC	**X1**	**X2**	**X1*X2**	**adhesion strength (MPa)**
1	-1	-1	+1	6.0
2	-1	+1	-1	7.4
3	+1	-1	-1	10.8
4	+1	+1	+1	11.2

a) Using ANalysis Of Means (ANOM), determine the effect of the curing agent, curing time, and the interaction between them on the adhesion strength. Which of these effects is most important?

b) Develop an equation to predict the adhesion strength as a function of the amount of curing agent (in mg) and curing time (in seconds). Be sure to include the interaction term in your model.

c) Use your equation to predict the average adhesion strength that would be developed with 70 mg of curing agent and a 30 second curing time.

d) What are the key assumptions you need to make in applying your equation in part c)?

3) A full-factorial design is being used to help with development of a prospective 3-D printing process for a ceramic scaffold material, in an attempt to improve the compressive strength. Three factors, each with two levels, have been tested, producing the results shown in the tables below.

Factor	Level	
	-1	+1
X1. Powder size (nm)	50	100
X2. Layer thickness (mm)	0.3	0.5
X3. Scan speed (mm/min)	100	300

TC	**X1**	**X2**	**X3**	**X1*X2**	**X1*X3**	**X2*X3**	**average compressive strength (MPa)**
1	-1	-1	-1				4.60
2	-1	-1	+1				2.30
3	-1	+1	-1				4.00
4	-1	+1	+1				1.90
5	+1	-1	-1				3.32
6	+1	-1	+1				1.38
7	+1	+1	-1				2.88
8	+1	+1	+1				1.22

a) Develop an equation to predict the average compressive strength as a function of these three factors. Include all two factor interactions (e.g. X1*X2) in your model, but not the three factor interaction (i.e. do not include an X1*X2*X3 term).
b) How many degrees of freedom are used in your equation/model in part a)?
c) Determine the best values of the three factors to maximize the compressive strength and predict the strength that might be expected. Assume that the factors must be kept with the range of values tested in the experiment (i.e. do not use the equation to extrapolate beyond the values tested).
d) What are the key assumptions you need to make in applying your equation in part c)?

Part 2.

4) A full-factorial experiment was conducted to study the effects of adding thickening agents to tea on its viscosity (for dysphagia patients). Pooling of interactions was used to provide an error estimate for use in ANOVA. The results are summarized in the table below.

Factor	Level	
	-1	+1
X1. thickening agent	"A"	"B"
X2. concentration (g/4 oz. liquid)	1	2
X3. mixing procedure	stir	shake

Critical F for α = 0.05

DOF (error)	DOF (effect)		
	1	2	3
1	161.45	199.50	215.71
2	18.51	19.00	19.16
3	10.13	9.55	9.28
4	7.71	6.94	6.59

Source	**SS**	**DOF**	**MS**	**F**	**% SS**
X1	3210	1			
X2	7270	1			
X3	210	1			
error	440	4		--	
Total	11130	7	--	--	100

a) Do the necessary calculations and complete the remaining columns on the ANOVA table.

b) Summarize your conclusions (significance, importance) based on the ANOVA (use α = 0.05).

5) The effects of the amount of curing agent and the curing time on the adhesion strength of dental brackets were studied using a full factorial experiment. The experiment was replicated to provide an error estimate and the results are summarized in the tables below.

Factor	Level	
	-1	+1
X1. amount of curing agent (mg)	50	100
X2. curing time (seconds)	15	60

Critical F for α = 0.05

DOF (error)	DOF (effect)		
	1	2	3
1	161.45	199.50	215.71
2	18.51	19.00	19.16
3	10.13	9.55	9.28
4	7.71	6.94	6.59

TC	**X1**	**X2**	**X1*X2**	**adhesion strength (MPa)**
1a	-1	-1	+1	6.0
2a	-1	+1	-1	7.4
3a	+1	-1	-1	10.8
4a	+1	+1	+1	11.2
1b	-1	-1	+1	5.3
2b	-1	+1	-1	7.4
3b	+1	-1	-1	10.8
4b	+1	+1	+1	11.7

a) Use ANalysis Of Means (ANOM), to determine the effect of the curing agent, curing time, and the interaction between them on the adhesion strength.
b) Use ANOVA to determine which of these effects are statistically significant (use α = 0.05).
c) Develop an equation to predict the adhesion strength. Include only statistically significant effects in the model.
d) Use your equation to predict the average adhesion strength that would be developed with 70 mg of curing agent and a 30 second curing time.
e) What is the key assumption you need to make in applying your equation in part d)?
f) How do the results of this analysis compare to those you obtained in question 2) from Part 1? Discuss.

6) A full factorial experimental study is to be conducted to study the effect of screw geometry (outer diameter, pitch, etc.) on the bone holding strength of screws used for tibial repair. Two-level factors are to be used and a list of six potential factors (X1 through X6) have been identified. However, it may not be possible to test all six factors in this study.

a) Assume that all treatment conditions are to be executed twice (i.e. replicated) to provide an error estimate. How many factors will it be possible to study if you are limited to testing a maximum of 32 screws? (Note: testing destroys a screw so it cannot be reused.)

b) For the experimental program in part a), how many degrees of freedom would be available for the error estimate and what would be the critical F value for judging the significance of a factor? [Notes: Assume that only "pure error", replication, will be used for the error estimate. Assume α = 0.05 to judge significance. You will need to look up a table of critical F values from a reliable online source or textbook.]

c) Now assume that the study will not be replicated (i.e. each treatment condition will be run only once). How many factors will it be possible to study in a full factorial design if you are again limited to testing a maximum of 32 screws? (Note: testing destroys a screw so it cannot be reused.)

d) For the experimental program in part c), an error estimate will be obtained by pooling the higher-order interactions. Assume that all interactions involving 4 or more factors are to be pooled. For this case, how many degrees of freedom would be available for the error estimate and what would be the critical F value for judging the significance of a factor? [Notes: Assume α = 0.05 to judge significance. You will need to look up a table of critical F values from a reliable online source or textbook.]

Part 3.

7) A full factorial experiment is to be used to build a model to predict water uptake in a PDMS polymer for use in cochlear-implant electrodes. (Note: Some details for this problem are taken from Mizdeh and Abbasi, Journal of Biomedical Materials Research - Part B, **68** (2004), 191-198). The two factors are concentration of a hydroxyethyl methacrylate monomer ("HEMA", variable over the range from 0.5 to 1.5 moles/liter) and the processing temperature (variable over the range 65 to 80 degrees C). Both linear and quadratic terms are to be included in the model.

Design a three level full factorial design for this experiment (i.e. construct a table showing each treatment condition with the settings to be used for each factor).

8) Zalnezhad et al. (Metallurgical and Materials Transactions A, 45A (2014), 785-797) used a fractional factorial design to study the effects of three processing factors on the adhesion strength of titanium/titanium oxide based coatings for biomedical applications. (Scratch force is taken as a measure of the adhesion strength, with a higher value being better.) The table below shows the factor settings and experimental results. Perform an ANOM analysis of this data. Discuss the relative importance of the three factors, the best settings for each to maximize the adhesion strength (scratch force), and whether any of the factors seem to have strong quadratic effects.

Treatment condition	DF Power (W)	Temperature (K)	DC Bias Voltage (V)	Scratch Force (mN)
1	200	426	25	876
2	200	473	50	1286
3	200	523	75	1390
4	300	426	50	1647
5	300	473	75	1185
6	300	523	25	1012
7	350	426	75	2395
8	350	473	25	1561
9	350	523	50	957

9) The following ANOVA tables are closely based on those in the cited articles. In each case, interpret the experimental results.

a) Thammarakchroen et al. (Journal of Nanoscience and Nanotechnology, 14 (2014), 7614-7620) used a Taguchi L18 design to study the effects of seven 3-level factors on the composition of biomimetic coatings on titanium. The table below is based on the ANOVA table presented in their paper, with some modification.

Factor no.	DOF	Sum of square	Mean square	F	Contribution (%)
#1	2	37.0	18.5	7.7	2.5
#2	2	50.5	25.2	10.5	3.4
#3	2	14.3	7.1	3.0	1.0
#4	2	115.3	57.6	24.0	7.7
#5	2	1118.3	559.1	233.0	74.9
#6	2	113.0	56.5	23.5	7.6
#7	2	38.4	19.2	8.0	2.6
Total	17	1494.0	-	-	100.0
Error	3	7.2	2.4	-	0.5

Critical F for $\alpha = 0.05$

DOF (error)	DOF (effect)		
	1	2	3
1	161.45	199.50	215.71
2	18.51	19.00	19.16
3	10.13	9.55	9.28
4	7.71	6.94	6.59

b) Franzetti et al. (International Biodeterioration and Biodegradation, 63 (2009), 943-947) used several different experimental designs to study the effect of eight 2-level factors on the production of biosurfactant by a hydrocarbon-deteriorating bacterium. Among their designs was a 64 treatment condition, fractional factorial, with four additional measurements added at the center point, for a grand total of 68 treatment conditions. The table below is based on the ANOVA table presented in their paper and shows only the largest effects.

Factor/interaction	SS	df	MS	F	p
X7	15735.05	1	15735.05	44.49	<0.00001
X3	6488.30	1	6488.30	18.34	0.00016
X3 * X7	5825.51	1	5825.51	16.46	0.00031
X6	3965.85	1	3965.85	11.21	0.00214
X6 * X7	3588.01	1	3588.01	10.14	0.00328
X3 * X6	2748.38	1	2748.38	7.77	0.00898
ERROR	10963.16	31	353.65	-	-
TOTAL	60085.58	67	-	-	-

c) As part of their follow-up to the experiments described in b), Franzetti et al. also conducted a Central-Composite Design (CCD) experiment on three factors (X3, X6, and X7) using a narrower range of factor values (centered around the conditions of most interest). The table below is based on the ANOVA table presented in their paper.

Factor/interaction	SS	df	MS	F	p
X3 (linear)	48.02	1	48.02	1.44	0.27542
X3 (quadratic)	65.44	1	65.44	1.96	0.21084
X6 (linear)	10.54	1	10.54	0.31	0.59432
X6 (quadratic)	52.25	1	52.25	1.56	0.25729
X7 (linear)	55.74	1	55.74	1.67	0.24366
X7 (quadratic)	46.21	1	46.21	1.38	0.28373
X3 (linear)* X6 (linear)	15.68	1	15.68	0.47	0.51855
X3 (linear)* X7 (linear)	14.04	1	14.04	0.42	0.54044
X6 (linear)* X7 (linear)	160.20	1	160.20	4.80	0.07092
ERROR	200.14	6	33.35	-	-
TOTAL	594.39	15	-	-	-

d) Lin et al. (Journal of Biomechanics, 43 (2010), 2174-2181) used a Taguchi L18 design to study the effects of design eight factors (one 2-level and seven 3-level) on maximum strain predicted in a finite element model of orthodontic screws. The table below is based on the ANOVA table presented in their paper, with some modification.

Design factor	d.o.f.	SS	MS	TSS (%)
Osseointegration	1	0	0	0
Screw length	2	16	8	1
Screw diameter	2	101	50	7
Thread shape	2	30	15	2
Thread depth	2	20	10	1
Screw material	2	917	458	63
Head diameter	2	33	16	2
Head exposure length	2	353	176	24
error	2	3	1	0
Total	17	1472	-	100