DOE module: Practice problem solutions

Part 1.

1) People with dysphagia can have difficulty swallowing liquids with too low a viscosity. To avoid this, “thickening agents” can be added to drinks to increase their viscosity. Assume that you have been assigned to study the effect of three factors on the thickness (viscosity) of tea. You have two possible thickening agents (“A” and “B”), two possible concentrations (1 or 2 grams in a 4 oz. drink), and two possible mixing procedures (stir and shake).

a) Which of these factors are discrete and which are continuous?

The thickening agent and mixing procedure are discrete factors (two distinct levels), while concentration is continuous (can have any value).

b) Design a full factorial experiment to measure the effect of these three factors on viscosity. In your design include columns to aid in calculating the two-factor and three-factor interaction effects.

A full factorial experiment should include all possible combinations of the three factors. An appropriate design is shown below:

Treatment condition	Thickening agent	Concentration (g per 4 oz.)	Mixing procedure
1	A	1	stir
2	A	1	shake
3	A	2	stir
4	A	2	shake
5	B	1	stir
6	B	1	shake
7	B	2	stir
8	B	2	shake

c) How many degrees of freedom are in your design?

Eight. (One for each treatment condition.)

2) The effects of the amount of curing agent and the curing time on the adhesion strength of dental brackets were studied using a full factorial experiment. The experiment and its results are summarized in the tables below.

Factor	**Level**	
	-1	+1
X1. amount of curing agent (mg)	50	100
X2. curing time (seconds)	15	60

TC	**X1**	**X2**	**X1*X2**	**adhesion strength (MPa)**
1	-1	-1	+1	6.0
2	-1	+1	-1	7.4
3	+1	-1	-1	10.8
4	+1	+1	+1	11.2

a) Using ANalysis Of Means (ANOM), determine the effect of the curing agent, curing time, and the interaction between them on the adhesion strength. Which of these effects is most important?

To complete the ANOM we average the results for all treatment conditions done at the same level. So, for example, to obtain the value for factor X1 at level +1, we average together the results for treatment conditions 3 and 4 [(10.8 + 11.2)/2 = 11.0]. We calculate the effect, Δ, by subtracting the "-1" level from the "+1" level (for example, for X1, Δ_{X1} = 11.0 – 6.7 = +4.3).

Factor/Interaction	Level		Δ
	-1	+1	
X1	6.7	11.0	+4.3
X2	8.4	9.3	+0.9
X1*X2	9.1	8.6	-0.5

Factor X1 has the largest effect (biggest difference between the two levels).

b) Develop an equation to predict the adhesion strength as a function of the amount of curing agent (in mg) and curing time (in seconds). Be sure to include the interaction term in your model.

First develop the equation based on the level (-1 to +1) coding:

$$y_{pred} = a_o + a_1X_1 + a_2X_2 + a_{12}X_1X_2$$

$$a_0 = m^* = (6.0 + 7.4 + 10.8 + 11.2)/4 = 8.85$$

$$a_1 = \Delta_{X1}/2 = (11.0 - 6.7)/2 = 2.15$$

$$a_2 = \Delta_{X2}/2 = (9.3 - 8.4)/2 = 0.45$$

$$a_{12} = \Delta_{X12}/2 = (8.6 - 9.1)/2 = -0.25$$

$$y_{pred} = 8.85 + (2.15)X_1 + (0.45)X_2 - (0.25)X_1X_2$$

Next convert to the actual parameters.

AC = amount of curing agent (mg) = 75 + (25) X1 --> $X1 = \frac{AC}{25} - 3$

CT = curing time (s) = 37.5 + (22.5) X2 --> $X2 = \frac{CT}{22.5} - 1.67$

$y_{pred} = 8.85 + 2.15\left(\frac{AC}{25} - 3\right) + \ 0.45\left(\frac{CT}{22.5} - 1.67\right) - \ 0.25\left(\frac{AC}{25} - 3\right)\left(\frac{CT}{22.5} - 1.67\right)$

Then do the algebra to simplify:

$$y_{pred} = 0.396 + (0.103)AC + (0.0533)CT - (0.000444)(AC * CT)$$

c) Use your equation to predict the average adhesion strength that would be developed with 70 mg of curing agent and a 30 second curing time.

$$y_{pred} = 0.396 + (0.103)(70) + (0.0533)(30) - (0.000444)(70 * 30)$$

$$= 8.27\ MPa$$

d) What are the key assumptions you need to make in applying your equation in part c)?

* That the effects being observed are real (not due to experimental error).
* That the effects are linear over the range tested.

3) A full-factorial design is being used to help with development of a prospective 3-D printing process for a ceramic scaffold material, in an attempt to improve the compressive strength. Three factors, each with two levels, have been tested, producing the results shown in the tables below.

Factor	**Level**	
	-1	+1
X1. Powder size (nm)	50	100
X2. Layer thickness (mm)	0.3	0.5
X3. Scan speed (mm/min)	100	300

TC	**X1**	**X2**	**X3**	**X1*X2**	**X1*X3**	**X2*X3**	**average compressive strength (MPa)**	**predicted strength**
1	-1	-1	-1	+1	+1	+1	4.60	4.61
2	-1	-1	+1	+1	-1	-1	2.30	2.29
3	-1	+1	-1	-1	+1	-1	4.00	3.99
4	-1	+1	+1	-1	-1	+1	1.90	1.91
5	+1	-1	-1	-1	-1	+1	3.32	3.31
6	+1	-1	+1	-1	+1	-1	1.38	1.39
7	+1	+1	-1	+1	-1	-1	2.88	2.89
8	+1	+1	+1	+1	+1	+1	1.22	1.21

a) Develop an equation to predict the average compressive strength as a function of these three factors. Include all two factor interactions (e.g. X1*X2) in your model, but not the three factor interaction (i.e. do not include an X1*X2*X3 term).
To help calculate the interaction terms, we complete the appropriate columns above based on the products. For example, the value of X1X2 for each treatment condition is given by the multiplying the values from the X1 and X2 columns for the same treatment condition.

To determine the constants for our equation, we first do an ANOM to find the values for each factor/interaction we average the results for all treatment conditions done at the same level. So, for example, to obtain the value for factor X1 at level +1, we average together the results for treatment conditions 5, 6, 7, and 8 [(3.32 + 1.38 + 2.88 + 1.22)/4 = 2.2]. These values are shown in the table below. The Δ value is then calculated as the difference between the +1 level and -1 level values. Finally, we divide Δ by 2 [= (+1) – (-1)] to get the slopes (a) for the equation. Finally, the constant a_0 = m* = overall average for the experiment = 2.70

Factor/Interaction	Level		Δ	a
	-1	+1		
X1	3.20	2.20	-1	-0.5
X2	2.90	2.50	-0.4	-0.2
X3	3.70	1.70	-2	-1
X1*X2	2.65	2.75	0.1	0.05
X1*X3	2.60	2.80	0.2	0.1
X2*X3	2.64	2.76	0.12	0.06

$$y_{pred} = a_o + a_1X_1 + a_2X_2 + a_3X_3 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{23}X_2X_3$$

$$\mathbf{y_{pred} = 2.7 - (0.5)X_1 - (0.2)X_2 - (1)X_3 + (0.05)X_1X_2 + (0.1)X_1X_3 - (0.06)X_2X_3}$$

b) How many degrees of freedom are used in your equation/model in part a)?

Seven, one for each of the constants in the equation.

c) Determine the best values of the three factors to maximize the compressive strength and predict the strength that might be expected. Assume that the factors must be kept with the range of values tested in the experiment (i.e. do not use the equation to extrapolate beyond the values tested).
Only linear effects, so the maximum should occur at the extreme (i.e. ±1) values for the factors. All three of the factor effects are quite a bit larger than the interactions, so it's reasonable to pick the values to maximize the factor contributions –> this means setting all three factors to -1. Putting -1 into the equation for all three factors (i.e. X1 = X2 = X3 = -1), yields a prediction of 4.61 MPa.

Note: Identifying best values can get a little messy, especially if the interaction effects are more comparable to the factors in size. As a check of these selections, and of the model in general, a column showing the predicted values for each treatment condition has been added to the table showing the experimental results above. In this case the model predictions are extremely close to the experimental results, as might be expected since we are using an equation with 7 constants to fit 8 total data points.

d) What are the key assumptions you need to make in applying your equation in part c)?

* That the effects being observed are real (not due to experimental error).
* That the effects are linear over the range tested.

Part 2.

4) A full-factorial experiment was conducted to study the effects of adding thickening agents to tea on its viscosity (for dysphagia patients). Pooling of interactions was used to provide an error estimate for use in ANOVA. The results are summarized in the table below.

Factor	**Level**	
	-1	+1
X1. thickening agent	"A"	"B"
X2. concentration (g/4 oz. liquid)	1	2
X3. mixing procedure	stir	shake

Critical F for α = 0.05

DOF (error)	DOF (effect)		
	1	2	3
1	161.45	199.50	215.71
2	18.51	19.00	19.16
3	10.13	9.55	9.28
4	7.71	6.94	6.59

Source	**SS**	**DOF**	**MS**	**F**	**% SS**
X1	3210	1	3210	29.2	28.9
X2	7270	1	7270	66.1	65.3
X3	210	1	210	1.9	1.9
error	440	4	110	--	4.0
Total	11130	7	--	--	100

a) Do the necessary calculations and complete the remaining columns on the ANOVA table.

Values have been added to the table above.

MS = SS/DOF; F = MS(factor)/MS(error); %SS = SS(factor)/SS(Total)

b) Summarize your conclusions (significance, importance) based on the ANOVA (use α = 0.05).

From the table, the critical F is 7.71.

$F > F_{critical}$ for factors X1 and X2 --> judged significant

$F < F_{critical}$ for factor X3 --> not judged significant

5) The effects of the amount of curing agent and the curing time on the adhesion strength of dental brackets were studied using a full factorial experiment. The experiment was replicated to provide an error estimate and the results are summarized in the tables below.

Factor	Level	
	-1	+1
X1. amount of curing agent (mg)	50	100
X2. curing time (seconds)	15	60

Critical F for $\alpha = 0.05$

DOF (error)	DOF (effect)		
	1	2	3
1	161.45	199.50	215.71
2	18.51	19.00	19.16
3	10.13	9.55	9.28
4	7.71	6.94	6.59

TC	X1	X2	X1*X2	adhesion strength (MPa)
1a	-1	-1	+1	6.0
2a	-1	+1	-1	7.4
3a	+1	-1	-1	10.8
4a	+1	+1	+1	11.2
1b	-1	-1	+1	5.3
2b	-1	+1	-1	7.4
3b	+1	-1	-1	10.8
4b	+1	+1	+1	11.7

a) Use ANalysis Of Means (ANOM), to determine the effect of the curing agent, curing time, and the interaction between them on the adhesion strength.

To complete the ANOM we average the results for all treatment conditions done at the same level. So, for example, to obtain the value for factor X1 at level +1, we average together the results for treatment conditions 3a, 3b, 4a, and 4b [(10.8 + 11.2 + 10.8 + 11.7)/4 = 11.125]. We calculate the effect, Δ, by subtracting the "-1" level from the "+1" level (for example, for X1, Δ_{X1} = 11.125 – 6.525 = +4.60).

Factor/Interaction	Level		Δ
	-1	+1	
X1	6.525	11.125	+4.60
X2	8.225	9.425	+1.20
X1*X2	9.100	8.550	-0.55

b) Use ANOVA to determine which of these effects are statistically significant (use $\alpha = 0.05$).

Example calculations

$m^* = \Sigma y_i / n = (6.0 + 7.4 + \ldots + 11.7)/8 = 8.825$

SS (for X1) = $\Delta^2 \times$ (# of TC)/4 = $(4.60)^2 \times (8)/4 = 42.32$

SS (Total) = $\Sigma(y_i - m^*)^2 = (6.0 - 8.825)^2 + (7.4 - 8.825)^2 + \ldots + (11.1 - 8.825)^2 = 46.175$

SS (error) = SS(Total) - ΣSS (factors and interactions) = 0.370

MS (for error) = SS (error)/DOF(error) = 0.370/4 = 0.0925

F (for X1) = MS (for X1)/MS (for error) = 42.32/0.0925 = 457.5

Source	SS	DOF	MS	F
X1	42.320	1	42.320	457.5
X2	2.880	1	2.880	31.1
X1*X2	0.605	1	0.605	6.5
error	0.370	4	0.0925	--
Total	46.175	7	--	--

$F_{critical}$ = 7.71 (1 DOF for factor interaction, 4 DOF for error, α = 0.05).
Therefore, judge the X1 and X2 effects to be significant. X1*X2 interaction not significant.

c) Develop an equation to predict the adhesion strength. Include only statistically significant effects in the model.

$y_{pred} = a_o + a_1X_1 + a_2X_2$

$a_0 = m^* = 8.825$

$a_1 = \Delta_{X1}/2 = 4.60/2 = 2.3$

$a_2 = \Delta_{X2}/2 = 1.20/2 = 0.6$

$y_{pred} = 8.825 + (2.3)X_1 + (0.6)X_2$

Next convert to the actual parameters.

AC = amount of curing agent (mg) = 75 + (25) X1 --> $X1 = \frac{AC}{25} - 3$

CT = curing time (s) = 37.5 + (22.5) X2 --> $X2 = \frac{CT}{22.5} - 1.67$

$$y_{pred} = 8.825 + 2.3\left(\frac{AC}{25} - 3\right) + 0.6\left(\frac{CT}{22.5} - 1.67\right)$$

Then do the algebra to simplify:

$$y_{pred} = 0.923 + (0.092)AC + (0.0267)CT$$

d) Use your equation to predict the average adhesion strength that would be developed with 70 mg of curing agent and a 30 second curing time.

$$y_{pred} = 0.923 + (0.092)(70) + (0.0267)(30)$$

$$= 8.16\ MPa$$

e) What is the key assumption you need to make in applying your equation in part d)?
* That the effects are linear over the range tested.

f) How do the results of this analysis compare to those you obtained in question 2) from Part 1? Discuss.
The results are quite similar. This reflects the fact that the dominant terms in question 2 turn out to be significant and, thus, are also included in the model developed here. The main difference in the model is just the removal of the small interaction term.

If this were not the case (i.e. if X2, or even X1, were judged not significant) then the models would be very different. What's gained by the replication, therefore, is the increased confidence that the dominant terms are significant and should be included in the model.

6) A full factorial experimental study is to be conducted to study the effect of screw geometry (outer diameter, pitch, etc.) on the bone holding strength of screws used for tibial repair. Two-level factors are to be used and a list of six potential factors (X1 through X6) have been identified. However, it may not be possible to test all six factors in this study.

a) Assume that all treatment conditions are to be executed twice (i.e. replicated) to provide an error estimate. How many factors will it be possible to study if you are limited to testing a maximum of 32 screws? (Note: testing destroys a screw so it cannot be reused.)
Replicating reduces the number of distinct combinations of factors we can test by a factor of two. So we are limited to 16 combinations, which corresponds to all possible combinations of <u>four</u> 2-level factors ($16 = 2^4$).

b) For the experimental program in part a), how many degrees of freedom would be available for the error estimate and what would be the critical F value for judging the significance of a factor? [Notes: Assume that only "pure error", replication, will be used for the error estimate. Assume $\alpha = 0.05$ to judge significance. You will need to look up a table of critical F values online or in a textbook.]
The factors and interactions will all be two level (either -1 or +1), so there is 1 DOF for each of the effects.
There are 16 DOF for pure error (one for each of the replicated treatment conditions.
With $\alpha = 0.05$ we obtain: $F_{critical} = 4.49$

c) Now assume that the study will not be replicated (i.e. each treatment condition will be run only once). How many factors will it be possible to study in a full factorial design if you are again limited to testing a maximum of 32 screws? (Note: testing destroys a screw so it cannot be reused.)
In this case we have 32 possible combinations, which corresponds to all possible combinations of <u>five</u> 2-level factors ($32 = 2^5$).

d) For the experimental program in part c), an error estimate will be obtained by pooling the higher-order interactions. Assume that all interactions involving 4 or more factors are to be pooled. For this case, how many degrees of freedom would be available for the error estimate and what would be the critical F value for judging the significance of a factor? [Notes: Assume $\alpha = 0.05$ to judge significance. You will need to look up a table of critical F values online or in a textbook.]
Interactions to be assumed zero (used for error estimate):
5-factor, 1 DOF, (X1*X2*X3*X4*X5)
4-factor, 5 DOF, (X1*X2*X3*X4, X1*X2*X3*X5, X1*X2*X4*X5, X1*X3*X4*X5, X2*X3*X4*X5)
Yields a total of **5 DOF** for error estimate. 1 DOF for factors and interactions. With $\alpha = 0.05$:

$$F_{critical} = 6.61$$

Part 3.

7) A full factorial experiment is to be used to build a model to predict water uptake in a PDMS polymer for use in cochlear-implant electrodes. (Note: Some details for this problem are taken from Mizdeh and Abbasi, Journal of Biomedical Materials Research - Part B, **68** (2004), 191-198). The two factors are concentration of a hydroxyethyl methacrylate monomer ("HEMA", variable over the range from 0.5 to 1.5 moles/liter) and the processing temperature (variable over the range 65 to 80 degrees C). Both linear and quadratic terms are to be included in the model.

Design a three level full factorial design for this experiment (i.e. construct a table showing each treatment condition with the settings to be used for each factor).

As much as possible we want to evenly space the factor levels, so that the linear and quadratic effects will be clearly distinguished. This yields values of 0.5, 1.0, and 1.5 moles/liter for HEMA, and 65, 73, and 80 degrees C for the temperature. Full factorial includes all possible combinations. Thus:

Treatment condition	HEMA concentration (moles/liter)	Temperature (degrees C)
1	0.5	65
2	0.5	73
3	0.5	80
4	1.0	65
5	1.0	73
6	1.0	80
7	1.5	65
8	1.5	73
9	1.5	80

Note on source material: The study by Mizdeh and Abbasi used a fractional factorial experiment with four factors. This problem uses two of their factors to provide practice putting together a 3-level full factorial experiment.

8) Zalnezhad et al. (Metallurgical and Materials Transactions A, 45A (2014), 785-797) used a fractional factorial design to study the effects of three processing factors on the adhesion strength of titanium/titanium oxide based coatings for biomedical applications. (Scratch force is taken as a measure of the adhesion strength, with a higher value being better.) The table below shows the factor settings and experimental results. Perform an ANOM analysis of this data. Discuss the relative importance of the three factors, the best settings for each to maximize the adhesion strength (scratch force), and whether any of the factors seem to have strong quadratic effects.

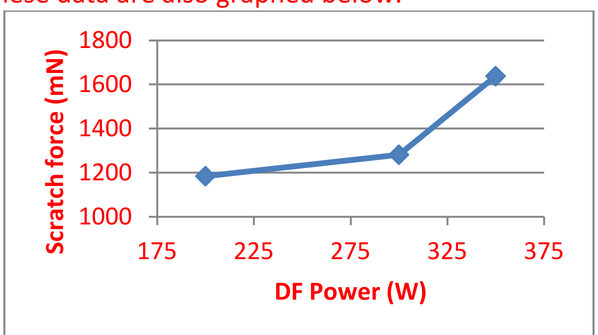
Treatment condition	DF Power (W)	Temperature (K)	DC Bias Voltage (V)	Scratch Force (mN)
1	200	426	25	876
2	200	473	50	1286
3	200	523	75	1390
4	300	426	50	1647
5	300	473	75	1185
6	300	523	25	1012

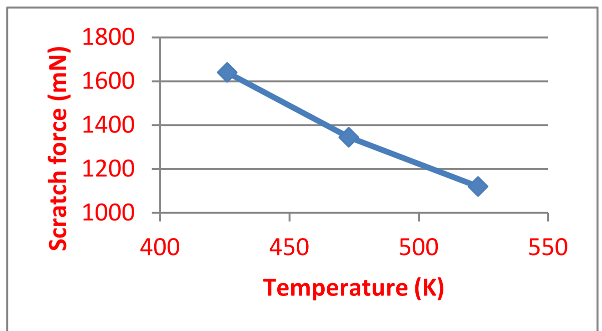
7	350	426	75	2395
8	350	473	25	1561
9	350	523	50	957

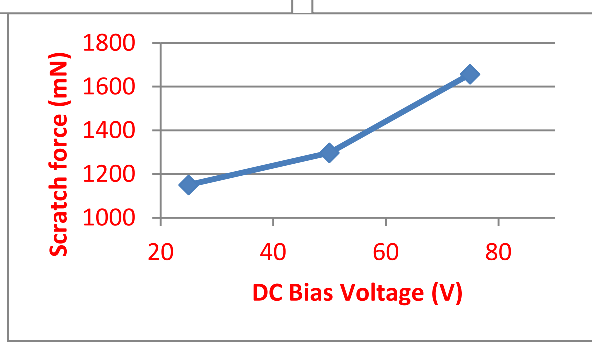
First do ANOM

Factor	Level		
	Low	Medium	High
DF Power (200/300/350 W)	1184.0	1281.3	1637.7
Temperature (426/473/523 K)	1639.3	1344.0	1119.7
DC Bias Voltage (25/50/75 V)	1149.7	1296.7	1656.7

These data are also graphed below.







The range of values produced for the three factors over their levels is similar, so the ANOM suggests they are roughly equal in importance.
The best settings to maximize the scratch force are just those that give the maximum values in the ANOM. These are 350 W, 426 K, and 75V.
Examining the graphs, both the temperature and the DC bias voltage appear to be strongly linear, with little sign of a quadratic dependence. The DF power shows some evidence of a quadratic effect, although distinguishing this is complicated somewhat by the uneven spacing of the levels (200/300/350).

Note on source material: Zalnezhad et al. performed a mathematical transform on their scratch force data to obtain a Taguchi “S/N ratio” prior to doing an ANOM. In this problem we have used the raw scratch force data. The end results/conclusions from the analysis are similar.

9) The following ANOVA tables are closely based on those in the cited articles. In each case, interpret the experimental results.

a) Thammarakchroen et al. (Journal of Nanoscience and Nanotechnology, 14 (2014), 7614-7620) used a Taguchi L18 design to study the effects of seven 3-level factors on the composition of biomimetic coatings on titanium. The table below is based on the ANOVA table presented in their paper, with some modification.

Factor no.	DOF	Sum of square	Mean square	F	Contribution (%)
#1	2	37.0	18.5	7.7	2.5
#2	2	50.5	25.2	10.5	3.4
#3	2	14.3	7.1	3.0	1.0
#4	2	115.3	57.6	24.0	7.7
#5	2	1118.3	559.1	233.0	74.9
#6	2	113.0	56.5	23.5	7.6
#7	2	38.4	19.2	8.0	2.6
Total	17	1494.0	-	-	100.0
Error	3	7.2	2.4	-	0.5

Critical F for $\alpha = 0.05$

DOF (error)	DOF (effect) 1	2	3
1	161.45	199.50	215.71
2	18.51	19.00	19.16
3	10.13	9.55	9.28
4	7.71	6.94	6.59

We will use $\alpha = 0.05$ (this is somewhat arbitrary, but a common choice).
Thus $F_{cr} = 9.55$. Hence you can conclude that factor #2, 4, 5, and 6 are statistically significant.
Of these, #5 is by far the most dominant (~ 75% of total SS).
#4 and #6 contribute much less at ~ 7.5% each. But together #4, 5, and 6 account for ~ 90% of the total SS.
#2, although judged significant contributes only 3.4%.

Note on source material: Thammarakchroen et al. focused primarily on identifying the largest factor effects and comparing these with all other sources in the experiment. In line with this, they ultimately added the contributions from the smaller effects into the "error" term in their published ANOVA table. [This is a fairly common practice in the literature, but does not yield an error estimate appropriate for judging statistical significance.] For this problem, to provide an error estimate more suitable for judging significance, we have modified the ANOVA table in Thammarakchroen et al. by removing the factor effects from the error estimate, and recalculating the F values based on this.

b) Franzetti et al. (International Biodeterioration and Biodegradation, 63 (2009), 943-947) used several different experimental designs to study the effect of eight 2-level factors on the production of biosurfactant by a hydrocarbon-deteriorating bacterium. Among their designs was a 64 treatment condition, fractional factorial, with four additional measurements added at the center point, for a grand total of 68 treatment conditions. The table below is based on the ANOVA table presented in their paper and shows only the largest effects.

Factor/interaction	SS	df	MS	F	p
X7	15735.05	1	15735.05	44.49	<0.00001

X3	6488.30	1	6488.30	18.34	0.00016
X3 * X7	5825.51	1	5825.51	16.46	0.00031
X6	3965.85	1	3965.85	11.21	0.00214
X6 * X7	3588.01	1	3588.01	10.14	0.00328
X3 * X6	2748.38	1	2748.38	7.77	0.00898
ERROR	10963.16	31	353.65	-	-
TOTAL	60085.58	67	-	-	-

We will use $\alpha = 0.05$ (this is somewhat arbitrary, but a common choice).
Thus $F_{cr} = 4.17$. Hence you can conclude that <u>all</u> of the factors and interactions shown are statistically significant. [Alternatively, can also see this from the fact that $p < \alpha$].

Note on source material: This ANOVA table is taken directly from that in Franzetti et al. There are 30 factor and 2-factor interaction terms not shown in the table; it is assumed that this is because they were not significant. The error term includes contributions from both of the techniques we have discussed, “replication” (at the center point) and “pooling of higher order interactions”.

c) As part of their follow-up to the experiments described in b), Franzetti et al. also conducted a Central-Composite Design (CCD) experiment on three factors (X3, X6, and X7) using a narrower range of factor values (centered around the conditions of most interest). The table below is based on the ANOVA table presented in their paper.

Factor/interaction	SS	df	MS	F	p
X3 (linear)	48.02	1	48.02	1.44	0.27542
X3 (quadratic)	65.44	1	65.44	1.96	0.21084
X6 (linear)	10.54	1	10.54	0.31	0.59432
X6 (quadratic)	52.25	1	52.25	1.56	0.25729
X7 (linear)	55.74	1	55.74	1.67	0.24366
X7 (quadratic)	46.21	1	46.21	1.38	0.28373
X3 (linear)* X6 (linear)	15.68	1	15.68	0.47	0.51855
X3 (linear)* X7 (linear)	14.04	1	14.04	0.42	0.54044
X6 (linear)* X7 (linear)	160.20	1	160.20	4.80	0.07092
ERROR	200.14	6	33.35	-	-
TOTAL	594.39	15	-	-	-

We will use $\alpha = 0.05$ (this is somewhat arbitrary, but a common choice).
Thus $F_{cr} = 5.99$. Hence you can conclude that <u>none</u> of the factors and interactions shown are statistically significant. [Alternatively, can also see this from the fact that $p > \alpha$].

Note on source material: This ANOVA table is also taken directly from that in Franzetti et al. Notice that in this experiment none of the factors had significant effects, while in the previous experiment all three of the same factors did. This result may at first seem contradictory, but it helps illustrate the point that just because something is not judged significant doesn't mean it doesn't have an effect. Judging statistical significance depends on our ability to clearly discern an effect against the background of error in the experiment and analysis. Things like the average value selected for a factor, the range of values used, and the settings for other factors can all influence whether an effect is seen clearly enough to be judged significant. In this particular case, the smaller range of values studied in the CCD is one likely contributor to the difference in the significance of the effects between the two experiments.

d) Lin et al. (Journal of Biomechanics, <u>43</u> (2010), 2174-2181) used a Taguchi L18 design to study the effects of design eight factors (one 2-level and seven 3-level) on maximum strain predicted in a finite element model of orthodontic screws. The table below is based on the ANOVA table presented in their paper, with some modification.

Design factor	d.o.f.	SS	MS	TSS (%)
Osseointegration	1	0	0	0
Screw length	2	16	8	1
Screw diameter	2	101	50	7
Thread shape	2	30	15	2
Thread depth	2	20	10	1

Screw material	2	917	458	63
Head diameter	2	33	16	2
Head exposure length	2	353	176	24
error	2	3	1	0
Total	17	1472	-	100

In this case, the design is being used to examine the results of an FE model, so we are primarily focused on identifying the relative importance of the various factors not judging statistical significance. From the %SS it can be seen that Screw material, Head exposure length and Screw diameter (in that order) are the dominant factors, together accounting for about 94% of the total SS. The other factors all account for < 2% each. The "error" term, a result of modeling error, is also very small.

Note on source material: Lin et al. did not include the "error" in their ANOVA table and, as a result, their total DOF was only 15. We have calculated the error from the raw data in their paper and included it in the table above to avoid possible confusion about this.