Problem Set 5
ME 444: Continuum Mechanics
D. H. Kelley
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1. Given the flow

\[ x_1 = X_1 + 3X_2 \]
\[ x_2 = X_2 \]
\[ x_3 = X_3 \]

in the \( \{\hat{e}_1, \hat{e}_2, \hat{e}_3\} \) basis,
(a) Find the deformation gradient \( F \).
(b) Find the right Cauchy-Green strain tensor \( C \).
(c) Find the eigenvalues and eigenvector of \( C \).
(d) Find the matrix of the stress tensor \( U \).
(e) Find its inverse \( U^{-1} \).
(f) Find the rotation tensor \( R \).
(g) Find the left Cauchy-green strain tensor \( B \).

2. Given the velocity field

\[ v_1 = k(x_2 - 2)^2x_3 \]
\[ v_2 = -x_1x_2 \]
\[ v_3 = kx_1x_3, \]

for an incompressible fluid in the \( \{\hat{e}_1, \hat{e}_2, \hat{e}_3\} \) basis, determine the value of the constant \( k \) such that the equation of mass conservation is satisfied.

3. Consider the deformation

\[ x_1 = 3X_3 \]
\[ x_2 = -X_1 \]
\[ x_3 = -2X_2 \]

in the \( \{\hat{e}_1, \hat{e}_2, \hat{e}_3\} \) basis.
(a) Find the ratio of deformed volume to initial volume.
(b) Find the rotation tensor \( R \).
(c) Find the left Cauchy-Green strain tensor \( B \).
(d) Find the stretch of a material element currently along the \( \hat{e}_3 + \hat{e}_1 \) direction.
(e) Find the initial angle between material elements currently along the \( \hat{e}_3 + \hat{e}_1 \) and \( 2\hat{e}_3 + \hat{e}_2 \) directions.

4. The stress distribution in a certain body is given by

\[ T = \begin{bmatrix}
0 & 100x_1 & -100x_2 \\
100x_1 & 0 & 0 \\
-100x_2 & 0 & 0
\end{bmatrix} \text{ MPa} \]

in Cartesian coordinates. Find the stress vector acting on a plane that passes through the point \( x = [1/2, \sqrt{3}/2, 3] \) and is tangent to the circular cylindrical surface \( x_1^2 + x_2^2 = 1 \).