Problem 1.4

1. Draw a free body diagram of the mass.

\[ \begin{align*}
   & O \\
   & \downarrow \theta \\
   & m \\
   & mg \sin \theta \\
   & mg \cos \theta \\
   & L \\
   & T
\end{align*} \]

2. Write equation of motion in tangential direction.

Method 1: By Newton's law.

\[-mg \sin \theta = ma\]
\[-mg \sin \theta = mL \ddot{\theta} \]
\[ml \ddot{\theta} + mg \sin \theta = 0 \]

(a)

This nonlinear differential equation governs the motion for any rotation \( \theta \).

Method 2: Equilibrium of moments about \( O \) yields

\[ ml^2 \ddot{\theta} = -mgL \sin \theta \]

or

\[ mL \ddot{\theta} + mg \sin \theta = 0 \]

3. Linearize for small \( \theta \).

For small \( \theta \), \( \sin \theta = \theta \), and Eq. (a) becomes

\[ ml \ddot{\theta} + mg \theta = 0 \]
\[ \ddot{\theta} + \left( \frac{g}{L} \right) \theta = 0 \]

(b)

4. Determine natural frequency.

\[ \omega_n = \sqrt{\frac{g}{L}} \]
Problem 1.5

1. Find the moment of inertia about O.
   
   From Appendix 8,
   
   \[ I_0 = \frac{1}{12} mL^2 + m \left( \frac{L}{2} \right)^2 = \frac{1}{3} mL^2 \]

2. Draw a free body diagram of the body in an arbitrary displaced position.

3. Write the equation of motion using Newton's second law of motion.

   \[ \sum M_O = I_0 \ddot{\theta} \]

   \[ -mg \frac{L}{2} \sin \theta = \frac{1}{3} mL^2 \ddot{\theta} \]

   \[ \frac{mL^2}{3} \dddot{\theta} + \frac{mgL}{2} \sin \theta = 0 \quad (a) \]

4. Specialize for small \( \theta \).

   For small \( \theta \), \( \sin \theta \equiv \theta \) and Eq. (a) becomes

   \[ \frac{mL^2}{3} \dddot{\theta} + \frac{mgL}{2} \theta = 0 \]

   \[ \dddot{\theta} + \frac{3g}{2L} \theta = 0 \quad (b) \]

5. Determine natural frequency.

   \[ \omega_n = \sqrt{\frac{3g}{2L}} \]
Problem 1.7

Draw a free body diagram of the mass:

\[ f_s \]
\[ \downarrow \]
\[ m \]
\[ \uparrow \]
\[ p(t) \]

Write equation of dynamic equilibrium:
\[ m\ddot{u} + f_s = p(t) \quad \text{(a)} \]

Write the force-displacement relation:
\[ f_s = \left( \frac{AE}{L} \right) u \quad \text{(b)} \]

Substitute Eq. (b) into Eq. (a) to obtain the equation of motion:
\[ m\ddot{u} + \left( \frac{AE}{L} \right) u = p(t) \]
**Problem 1.12**

2. **Determine the effective stiffness.**

\[ f_s = k_e \overline{\delta} \]  
\[ \text{(d)} \]

where

\[ \overline{\delta} = \delta_{\text{spring}} + \delta_{\text{beam}} \]  
\[ \text{(e)} \]

\[ f_s = k \delta_{\text{spring}} = k_{\text{beam}} \delta_{\text{beam}} \]  
\[ \text{(f)} \]

Substitute for the \( \delta \)'s from Eq. (f) and for \( \overline{\delta} \) from Eq. (d):

\[ f_s = f_s + f_s \]  
\[ k_e = k \]  
\[ k_{\text{beam}} \]  
\[ \text{(g)} \]

\[ k_e = \frac{kk_{\text{beam}}}{k + k_{\text{beam}}} \]  
\[ \text{(h)} \]

\[ k_e = \frac{k}{k + k_{\text{beam}}} \]  
\[ \text{(i)} \]

\[ k_e = \frac{48EI/L^3}{k + 48EI/L^3} \]  
\[ \text{(j)} \]

3. **Determine the natural frequency.**

\[ \omega_n = \sqrt{\frac{k_e}{m}} \]  
\[ \text{(k)} \]

1. **Write the equation of motion.**

Equilibrium of forces in Fig. 1.12c gives

\[ m \ddot{\overline{\delta}} + f_s = w + p(t) \]  
\[ \text{(a)} \]

where

\[ f_s = k_e \overline{\delta} \]  
\[ \text{(b)} \]

The equation of motion is:

\[ m \ddot{\overline{\delta}} + k_e \overline{\delta} = w + p(t) \]  
\[ \text{(c)} \]
Problem 1.14

1. Define degrees of freedom (DOF).

2. Reduced stiffness coefficients.

Since there are no external moments applied at the pinned supports, the following reduced stiffness coefficients are used for the columns.

Joint rotation:

Joint translation:

3. Form structural stiffness matrix.

4. Determine lateral stiffness.

The lateral stiffness \( k \) of the frame can be obtained by static condensation since there is no force acting on DOF 2 and 3:

First partition \( k \) as

where
\[ k_{ui} = \frac{EI_c}{h^3} \begin{bmatrix} 6 \end{bmatrix} \]
\[ k_{r0} = \frac{EI_c}{h^3} \begin{bmatrix} 3h & 3h \end{bmatrix} \]
\[ k_{00} = \frac{EI_c}{h^3} \begin{bmatrix} 5h^2 & h^2 \\ h^2 & 5h^2 \end{bmatrix} \]

Then compute the lateral stiffness \( k \) from
\[ k = k_{xz} - k_{r0}^{-1} k_{00} k_{r0} \]

Since
\[ k_{00}^{-1} = \frac{h}{24EI_c} \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} \]

we get
\[ k = \frac{6EI_c}{h^3} - \frac{EI_c}{h^3} \begin{bmatrix} 3h & 3h \end{bmatrix} \frac{h}{24EI_c} \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix} \frac{EI_c}{h^3} \begin{bmatrix} 3h \end{bmatrix} \]
\[ k = \frac{6EI_c}{h^3} - 3 \]
\[ k = \frac{3EI_c}{h^3} \]

5. Equation of motion.
\[ m \ddot{u} + \frac{3EI_c}{h^3} u = p(t) \]
Problem 2.1

Given:

\[ T_n = 2\pi \sqrt{\frac{m}{k}} = 0.5 \text{ sec} \quad (a) \]

\[ T_n' = 2\pi \sqrt{\frac{m + 50/g}{k}} = 0.75 \text{ sec} \quad (b) \]

1. Determine the weight of the table.

Taking the ratio of Eq. (b) to Eq. (a) and squaring the result gives

\[ \left( \frac{T_n'}{T_n} \right)^2 = \frac{m + 50/g}{m} \Rightarrow 1 + \frac{50}{mg} = \left( \frac{0.75}{0.5} \right)^2 = 2.25 \]

or

\[ mg = \frac{50}{1.25} = 40 \text{ lbs} \]

2. Determine the lateral stiffness of the table.

Substitute for m in Eq. (a) and solve for k:

\[ k = 16\pi^2 m = 16\pi^2 \left( \frac{40}{386} \right) = 16.4 \text{ lbs/in.} \]
Problem 2.3

1. Set up equation of motion.

\[ ku + mg/2 \]

\[ \begin{array}{c}
m\ddot{u} \\
u \\
mg 
\end{array} \]

\[ m\ddot{u} + ku = \frac{mg}{2} \]

2. Solve equation of motion.

\[ u(t) = A \cos \omega_d t + B \sin \omega_d t + \frac{mg}{2k} \]

At \( t = 0 \), \( u(0) = 0 \) and \( \dot{u}(0) = 0 \)

\[ \therefore A = -\frac{mg}{2k}, \quad B = 0 \]

\[ u(t) = \frac{mg}{2k} (1 - \cos \omega_d t) \]