Lecture 11: Pseudononholonomic constraints

We can take complicated (nonsimple) holonomic constraints and differentiate them to make them look like nonholonomic constraints

\[ f(q^i) = 0 \implies \frac{\partial f}{\partial q^i} \dot{q}^i \]

(remember the summation convention)
Current Euler-Lagrange ritual goes like this

Lagrangian

Holonomic constraints

Generalized coordinates

\[ \dot{q}^i = u^i \] velocity equation

Nonholonomic constraints/ Lagrange multipliers

momentum equations

Convert to time dependence/ simulate the system
Current Hamilton ritual goes like this

Lagrangian

Holonomic constraints

Generalized coordinates

Momentum $\rightarrow$ velocity equations

Nonholonomic constraints/ Lagrange multipliers

momentum equations

Convert to time dependence/ simulate the system
New ritual (based on Hamilton’s equations) goes like this

Lagrangian

Simple holonomic constraints

Generalized coordinates

Momentum $\rightarrow$ velocity equations

Nonsimple holonomic constraints $\rightarrow$ augmented nonholonomic constraints

Nonholonomic constraints

Lagrange multipliers

momentum equations

Convert to time dependence/ simulate the system
The differences

more generalized coordinates

more differential equations

more Lagrange multipliers

So, why would you want to do this!!
The equations are generally MUCH simpler
  (although this is not really true for this first illustrative case)

Lots of simple equations are generally better computationally
  than fewer, more complicated equations

Let me outline how it goes for the homework picture
  I’ll do it all three ways in Mathematica before today’s lecture is over
We have three links

There are eight simple holonomic constraints (orientation)

There are four nonsimple holonomic constraints (connectivity)

There are apparently four rolling constraints but they turn out not to be independent
\( q = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ \phi_1 \\ \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} \)
The nonsimple constraints

\[ q^1 - q^3 - \frac{1}{2} l_R \sin q^7 = 0 \]
\[ q^2 - q^4 + \frac{1}{2} l_R \sin q^7 = 0 \]
\[ q^1 - q^5 + \frac{1}{2} l_R \sin q^7 = 0 \]
\[ q^2 - q^6 - \frac{1}{2} l_R \sin q^7 = 0 \]

We’d solve these for \( q^3 \) through \( q^6 \) under the old rules
Differentiate them to get

\[ \dot{q}^1 - \dot{q}^3 - \frac{1}{2} l_R \cos q^7 \dot{q}^7 = 0 \]
\[ \dot{q}^2 - \dot{q}^4 + \frac{1}{2} l_R \cos q^7 \dot{q}^7 = 0 \]
\[ \dot{q}^1 - \dot{q}^5 + \frac{1}{2} l_R \cos q^7 \dot{q}^7 = 0 \]
\[ \dot{q}^2 - \dot{q}^6 - \frac{1}{2} l_R \cos q^7 \dot{q}^7 = 0 \]

linear in the derivatives
We’ll have four lines of a constraint matrix, to be added to that from the true nonholonomic constraints

\[
C = \begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & -\frac{1}{2} l_R \cos q^7 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & \frac{1}{2} l_R \cos q^7 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & \frac{1}{2} l_R \cos q^7 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & -\frac{1}{2} l_R \cos q^7 & 0 & 0 & 0
\end{bmatrix}
\]
Euler-Lagrange/Hamilton approaches have differential twelve equations

The new scheme has twenty

Euler-Lagrange/Hamilton has a 3 x 6 constraint matrix

The new scheme has a 7 x 10 constraint matrix
This is not a very good example to examine complexity

But it was a homework problem
and it’s good for review
so let’s look at this using all three methods.