ME 406 ASSIGNMENT #9

PROBLEMS DUE BY 6 PM ON FRIDAY APRIL 16, 2009

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
2.3 Local Behavior Near Equilibria	T,Th Apr 7, 9	---

There is not much in the text on this material. Useful references include Perko, sections 2.5-2.8, and Verhulst, Chapters 7 and 13.

PROBLEMS

(1) (10 points) For the third order system given below, show that the surface $x^2 + y + 3z = 0$ is an invariant manifold.

$$\dot{x} = -x \ , \dot{y} = x^2 + y \ , \dot{z} = \tfrac{2}{3}x^2 + z \quad .$$

(2) (30 points) For the second order system given below, show that the origin is a hyperbolic equilibrium point. Find analytically the stable and unstable manifolds for the linearized system, and use DynPac to find the stable and unstable manifolds for the nonlinear system. Combine the graphical results for the manifolds for the linear and nonlinear systems on a single plot.

$$\dot{x} = -x + y^2 \ \ , \dot{y} = x + y + xy \quad .$$

(3) (30 points) Find the stable and unstable manifolds for the linear system $\dot{\mathbf{X}} = \mathbf{AX}$, where the matrix $\mathbf{A}$ is given below. What is the most general initial condition which will decay to zero as t goes to infinity?

$$\mathbf{A} = \begin{pmatrix} 0 & \frac{1}{4} & \frac{3}{2} & \frac{3}{4} & -\frac{7}{4} \\ \frac{3}{2} & \frac{3}{4} & -\frac{1}{2} & -\frac{5}{4} & \frac{11}{4} \\ 1 & -\frac{1}{4} & -\frac{1}{2} & -\frac{3}{4} & \frac{7}{4} \\ 5 & \frac{19}{4} & -\frac{1}{2} & -\frac{15}{4} & \frac{19}{4} \\ \frac{5}{2} & \frac{5}{2} & 1 & -2 & \frac{5}{2} \end{pmatrix}$$

(4) (30 points. Problem 13-3, page 190, Verhulst). For the third order system given below, show that the origin is a non-hyperbolic equilibrium point. Find the stable and center manifolds for the linearized system. Find an approximation for the center manifold of the nonlinear system, and determine whether the equilibrium is stable or unstable. (Hint: In this case the center manifold will be described by $z = h(x,y)$. You should be able to derive a partial differential equation satisfied by h.)

$$\dot{x} = -y + xz - x^4 \ , \ \dot{y} = x + yz + xyz \ , \ \dot{z} = -z - x^2 - y^2 + z^2 + \sin(x^3) \quad .$$