# ME 406 ASSIGNMENT #8

**PROBLEMS DUE BY 6 PM ON FRIDAY APRIL 10, 2009**

## LECTURE SCHEDULE AND READING

| <u>Section in Class Notes</u> | <u>Date</u> | <u>Section in Text</u> |
|---|---|---|
| II. HIGHER ORDER AUTONOMOUS SYSTEMS | | |
| 2.1 Linear Systems – Distinct Eigenvalues | Th, T Mar 26, 31 | --- |
| 2.2 Linear Systems – Repeated Eigenvalues | Th Apr 2 | --- |

## PROBLEMS

In all of the problems below, you are to use eigenvalue-eigenvector techniques for the solution. The only exception is the last part of problem 4 in which you are to use DynPac. In carrying out the eigensystem analyses, you may use Mathematica in any way you like. In particular, you may use the notebook on these techniques which was handed out in class.

**(1)** (20 points) Consider the system $\dot{\mathbf{X}} = \mathbf{AX}$ where **A** is a constant *n* x *n* matrix. Let λ be an eigenvalue of **A** of multiplicity *m,* where $1 < m \le n$ . As discussed in class, **A** has *m* linearly independent generalized eigenvectors $\mathbf{V}^{(\mathbf{i})}$, *i*= 1, *m*. From each such eigenvector, we can form a solution of the differential equation. The form of the solution, as shown in class, is

$$\{\mathbf{I} + t(\mathbf{A} - \lambda\mathbf{I}) + \tfrac{t^2}{2!}(\mathbf{A} - \lambda\mathbf{I})^2 + \tfrac{t^3}{3!}(\mathbf{A} - \lambda\mathbf{I})^3 + \cdots + \tfrac{t^{m-1}}{(m-1)!}(\mathbf{A} - \lambda\mathbf{I})^{m-1}\}\mathbf{V}e^{\lambda t} \quad .$$

Show that the *m* solutions so formed are linearly independent.

**(2)** (25 points) Consider the system

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{b}, \text{ where } \mathbf{A} = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 4 & 0 \\ 2 & 1 & 4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \quad .$$

**(a)** Solve the initial value problem for

$$\mathbf{X}(0) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad .$$

**(b)** Solve the initial value problem for

$$\mathbf{X}(0) = \begin{pmatrix} -\frac{17}{24} \\ \frac{1}{4} \\ -\frac{5}{24} \end{pmatrix} \quad .$$

**(CONTINUED NEXT PAGE)**

**(3)** ( 25 points) Solve the initial value problem below.

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}\,,\ \text{with } \mathbf{X}(0) = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \text{where } \mathbf{A} = \begin{pmatrix} -1 & 4 & -1 \\ -2 & -2 & 2 \\ 1 & -4 & -3 \end{pmatrix} .$$

**(4)** (30 points) Consider the initial-value problem given below.

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X}, \text{ where } \mathbf{A} = \begin{pmatrix} 0 & \frac{1}{4} & \frac{3}{2} & \frac{3}{4} & -\frac{7}{4} \\ \frac{3}{2} & \frac{3}{4} & -\frac{1}{2} & -\frac{5}{4} & \frac{11}{4} \\ 1 & -\frac{1}{4} & -\frac{1}{2} & -\frac{3}{4} & \frac{7}{4} \\ 5 & \frac{19}{4} & -\frac{1}{2} & -\frac{15}{4} & \frac{19}{4} \\ \frac{5}{2} & \frac{5}{2} & 1 & -2 & \frac{5}{2} \end{pmatrix}, \text{ with } \mathbf{X}(0) = \begin{pmatrix} 1 \\ 0 \\ 2 \\ -1 \\ 1 \end{pmatrix} .$$

**(a)** Solve using the method of generalized eigenvectors. Plot the first component of your solution versus time for $0 \le t \le 2$ .

**(b)** Solve the problem using Dynpac, and again plot the first component versus time for $0 \le t \le 2$ .

**(c)** Compare the numerical values for the analytical and DynPac solutions at $t = 2$.