

ME 406 ASSIGNMENT #7

PROBLEMS DUE BY 6 PM ON FRIDAY MARCH 27, 2009

LECTURE SCHEDULE AND READING

<u>Section in Class Notes</u>	<u>Date</u>	<u>Section in Text</u>
I. PLANE AUTONOMOUS SYSTEMS		
1.11 Bifurcations	T,Th,T Mar 17, 19, 24	Chapt. 4, 8

PROBLEMS

(1) (50 points. (Problem 8.1.6 Strogatz) Consider the system $\dot{x} = y - 2x$, $\dot{y} = \mu + x^2 - y$.

- (a) Sketch the nullclines and discuss possible bifurcations on the basis of your sketch.
- (b) Find and classify the bifurcations that occur as μ varies.
- (c) Construct a sequence of phase portraits illustrating the bifurcations.

(2) (50 points. (This predator-prey model is given in problem 8.28 in Strogatz – see also the reference to the work of Odell given there.) Consider the predator-prey system given by

$$\dot{x} = x[x(1-x) - y], \dot{y} = y(x-a).$$

- (a) Find the equilibrium points and classify them.
- (b) Find all of the bifurcations in this system as a varies.
- (c) In part (b) you should have found a Hopf bifurcation at $a = 1/2$. Is it subcritical or supercritical? For $a = 0.45$, give a plot of the limit cycle and find its period. What is the period of the limit cycle as $a \rightarrow 0.5$?
- (d) Construct a relevant set of phase portraits for various values of a .