Lecture Schedule and Reading

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<td>I. PLANE AUTONOMOUS SYSTEMS</td>
<td>T, Th, T Mar 17, 19, 24</td>
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<td>1.11 Bifurcations</td>
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Problems

(1) (50 points. (Problem 8.1.6 Strogatz) Consider the system $\dot{x} = y - 2x$, $\dot{y} = \mu + x^2 - y$.

(a) Sketch the nullclines and discuss possible bifurcations on the basis of your sketch.

(b) Find and classify the bifurcations that occur as $\mu$ varies.

(c) Construct a sequence of phase portraits illustrating the bifurcations.

(2) (50 points. (This predator-prey model is given in problem 8.28 in Strogatz – see also the reference to the work of Odell given there.) Consider the predator-prey system given by $\dot{x} = x[x(1-x) - y]$, $\dot{y} = y(x - a)$.

(a) Find the equilibrium points and classify them.

(b) Find all of the bifurcations in this system as $a$ varies.

(c) In part (b) you should have found a Hopf bifurcation at $a = 1/2$. Is it subcritical or supercritical? For $a = 0.45$, give a plot of the limit cycle and find its period. What is the period of the limit cycle as $a \to 0.5$?

(d) Construct a relevant set of phase portraits for various values of $a$.  
