ME 406 ASSIGNMENT #6

PROBLEMS DUE IN CLASS ON THURSDAY MARCH 5, 2009

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
I. PLANE AUTONOMOUS SYSTEMS		
1.8 Case Study: van der Pol Oscillator	Th Feb 19	7.5
1.9 Searching for Periodic Solutions	T Feb. 24	7.2 – 7.4
1.10 Stability of Periodic Solutions	Th,T Feb. 26, Mar 3	---

PROBLEMS

(1) (15 points. Problem 29, Chapter 3, Jordan and Smith, 4th edition) Show that the systems given below have no periodic solutions.

(a) $\dot{x} = y$, $\dot{y} = 1 + x^2 - (1 - x)y$.

(b) $\dot{x} = (x-1)^3 + xy^2$, $\dot{y} = y + y^3$.

(c) $\dot{x} = 1 - x^3 + y^2$, $\dot{y} = 2xy$.

(2) (20 points. Problem 3 Chapter 11 in Jordan and Smith, 4th edition) Consider the system

$$\dot{x} = x + y - x^3 - 6xy^2 \ , \ \dot{y} = -x/2 + 2y - 8y^3 - x^2 y \ .$$

(a) Show that $x = y = 0$ is an unstable equilibrium point.

(b) Let $V = x^2 + 2y^2$. Show that there is a periodic orbit between $V = 0.5$ and $V = 1$.

(c) By exploring the solutions with DynPac, convince yourself (and me) that the periodic solution is a limit cycle.

(3) (15 points. Problem 3 Chapter 8, Jordan and Smith, 4th edition) Find all of the limit cycles for the system given below. Which cycles are stable?

$$\dot{x} = -y + x\sin(r)\ , \dot{y} = x + y\sin(r) \ \ , r = (x^2 + y^2)^{1/2} \ .$$

(4) (15 points) Show that the system given below has a unique limit cycle, and discuss the stability of this limit cycle.

$$\dot{x} = -y + x(1-r^2)^2\ , \dot{y} = x + y(1-r^2)^2 \ \ , r = (x^2 + y^2)^{1/2} \ .$$

(5) (15 points) Consider the Liénard equation given below. Use the theorem given in class to establish that this equation has a unique stable limit cycle. Use DynPac to construct the limit cycle and neighboring solutions and verify the stability of the limit cycle.

$$\ddot{x} + (x^2 - 3)\dot{x} + \frac{x + x^3}{1 + x^2} = 0 \ .$$

(6) (20 points) On the basis of what we have seen so far, one might be tempted to conjecture that when systems with damping have periodic solutions, they occur only as limit cycles (as with the van der Pol equation), and not in continuous families as in conservative systems. Comment on this conjecture after using DynPac to study the system $\ddot{x} + x\dot{x} + x^3 = 0$.