ME 406 ASSIGNMENT #3

PROBLEMS DUE IN CLASS ON THURSDAY FEB. 5, 2009

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
I. PLANE AUTONOMOUS SYSTEMS		
1.3 Equilibrium and Stability in Linear Systems	T, Th Jan. 27, 29	Chapter 5

In this assignment you will be using DynPac to construct some phase plane portraits for linear systems. You can do this on the basis of what you know from Tutorial 1, but you might want to look at Tutorial 9 which shows how the portrait can be constructed in a more automated fashion with the routine portrait. In Tutorial 10 the automation is carried one step further with a bifurcation routine which does several parameter cases with one command.

PROBLEMS

(1) (50 points) Consider the system given by

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & a \\ -1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} \quad .$$

For each parameter value given below, carry out the following analysis: (1) determine the stability of the equilibrium point; (2) find any straight line solutions; (3) find the solution to the initial value problem with initial conditions $x(0) = 1$, $y(0) = 1$. (4) use DynPac to find the solution of part (3) and to plot $x(t)$ and $y(t)$ for that solution; (5) use DynPac to produce a phase plane plot for a reasonable selection of initial conditions. Calculations (1)–(3) are to be done analytically, either by hand or with the use of Mathematica (you may for example use Dsolve).

(a) $a = -3$.

(b) $a = 3/4$.

(c) $a = 5$.

(2) (50 points) This is a modification of problem 5.3.5 in Strogatz, dealing with the Romeo and Juliet model discussed in section 5.3 of the book.

If Romeo and Juliet are romantic clones ($\dot{R} = aR + bJ$, $\dot{J} = bR + aJ$), should they expect boredom or bliss? Discuss how the outcome depends on a and b. Construct phase plane portraits for each of the three cases below, and interpret your results in each case.

(a) $a = 1, b = 2.$ **(b)** $a = 2, b = 1.$ **(c)** $a = -2, b = 1.$