ME 406 ASSIGNMENT #10

PROBLEMS DUE BY 6 PM FRIDAY APRIL 24 2009

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
2.4 Volume in Phase Space	T Apr 14	---
2.5 Lorenz Equations	T, Th Apr 14,16	9.2, 9.3

PROBLEMS (Each problem is worth 25 points)

(1) (a) Consider a system with a hyperbolic, strictly stable equilibrium point. Show that in some neighborhood of the equilibrium point the divergence of the slope vector is negative.

(b) A Hamiltonian system is a system of the form

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i} \text{ and } \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i},$$

where the $2n$ state variables are q_i and p_i with $i = 1,...,n$, and where H is a given function of the state variables. Show that a Hamiltonian system cannot have a stable hyperbolic equilibrium point.

(2) In this problem you are asked to make a study of the sensitive dependence on initial conditions exhibited by chaotic solutions of the Lorenz equations. Use the parameter values $\sigma = 10$, $b = 8/3$ and $r = 28$. Start with the initial condition $\{10, 10, 10\}$. Consider small variations from this condition (for example $\{10, 10+\varepsilon, 10\}$ where ε is small), and study how the time of divergence of the solutions depends on the initial separation.

(3) Use DynPac to study the solutions of the Lorenz equations for the parameter values $\sigma = 10$, $b = 8/3$ and $r = 160$.

(4) Consider the system given below. Use the parameter values $a = 0.2$ and $b = 0.2$. Study this system for various values of c, and give a summary of what you find. Try to find at least one chaotic solution and plot separately x, y, and z as functions of time for that solution. You should be able to find much that is interesting for values of c in the range $0 < c < 10$.

$$\frac{dx}{dt} = -y - z \quad , \quad \frac{dy}{dt} = x + ay \quad , \quad \frac{dz}{dt} = b + xz - cz \quad .$$