Balthazar van der Pol (1889-1959)

Picture from Modern Differential Equations, Martha L. Abell andJames P. Braselton, Saunders, 1996.

■ 2. The van der Pol Equation

The van der Pol equation, in what is now considered to be standard form, is given by

$$\ddot{x} + \mu\left(x^2 - 1\right)\dot{x} + x = 0 \ . \qquad (1)$$

We see that it is an oscillator with a linear spring force and a nonlinear damping force. In all that follows, we take $\mu > 0$. The time in the equation has been scaled so that the frequency associated with the spring force alone is unity. The damping force varies in an interesting way. For $|x| < 1$, the damping is actually negative and hence produces an amplification of the motion. For $|x| > 1$, there is true damping and the motion decays. These observations suggest the possibility of an oscillation, in which the system starts at small x, is driven to large x by the amplification, and is then damped back to small x. We will explore this possibility by using DynPac to construct orbits. We define the equation for DynPac, after converting it to the following system:

$$\dot{x} = y, \ \dot{y} = -x - \mu(x^2 - 1)y \ . \qquad (2)$$