ME 406
Bifurcations I
Bifurcations in a Simple LInear System

```
sysid
```

```
Mathematica  6.0.3, DynPac 11.01, 1/12/2009
```

```
intreset; plotreset;
```

1. Introduction

In this series of notebooks on bifurcations, we look at simple examples of common bifurcations. We follow in part the discussion given in Chapter 3 of **Nonlinear Dynamics and Chaos**, Steven Strogatz, Addison-Wesley, 1994. The examples in all of the notebooks include the following: saddle-node bifurcation, transcritical bifurcation, supercritical pitchfork bifurcation, subcritical pitchfork bifurcation, and Hopf bifurcation. In this first notebook, we look at a very simple linear system with a single parameters. As the parameter is changed, the equilibrium goes through the following sequence: saddle, stable node, stable spiral. We construct a movie of the change with the parameter of the system orbits.

2. Definition of the System

We consider the following system, depending on one parameter a:

$$\dot{x} = ay \ , \ \ \dot{y} = -x - 2y \ .$$

This system in general has a single equilibrium point at (0,0), the character of which depends on a. We begin our analysis by defining the system for DynPac

```
setstate[{x, y}]; setparm[{a}]; slopevec = {a y, -x - 2 y};
sysname = "Bifurcation in Linear System";
```

We give a name to the equilibrium point at the origin.

```
eq = {0, 0};
```

We also note that when $a = 0$, the system is degenerate, and the entire line $y = -x/2$ is an equilibrium line. This suggests that $a = 0$ is an important value in the bifurcation study. We get the eigenvalues at the equilibrium.

```
eigsys[eq]
```

$$\left\{\left\{-1-\sqrt{1-a}\,,\,-1+\sqrt{1-a}\right\},\,\left\{\left\{-1+\sqrt{1-a}\,,\,1\right\},\,\left\{-1-\sqrt{1-a}\,,\,1\right\}\right\}\right\}$$

From the expressions for the eigenvalues, we conclude the following: for $a < 0$, the equilibrium is a saddle; for $0 < a < 1$, the equilibrium is a stable node; for $1 < a$, the equilibrium is a stable spiral. Thus the important bifurcations are at $a = 0$ (transition between saddle and node), and $a = 1$ (transition between node and spiral). The most important bifurcation is at $a = 0$ because it is a stability transition. We now construct a sequence of phase plane plots showing the bifurcations. Each plot shows a representative set of integral curves for a given value of a. The connection between the plots is that the integral curves are always anchored at the same spots on the edges of the plotting window, and a specific curve color is associated with each anchor spot. On the graphs, we place a black dot at each of the anchor points. We set the parameters for integrating and plotting.

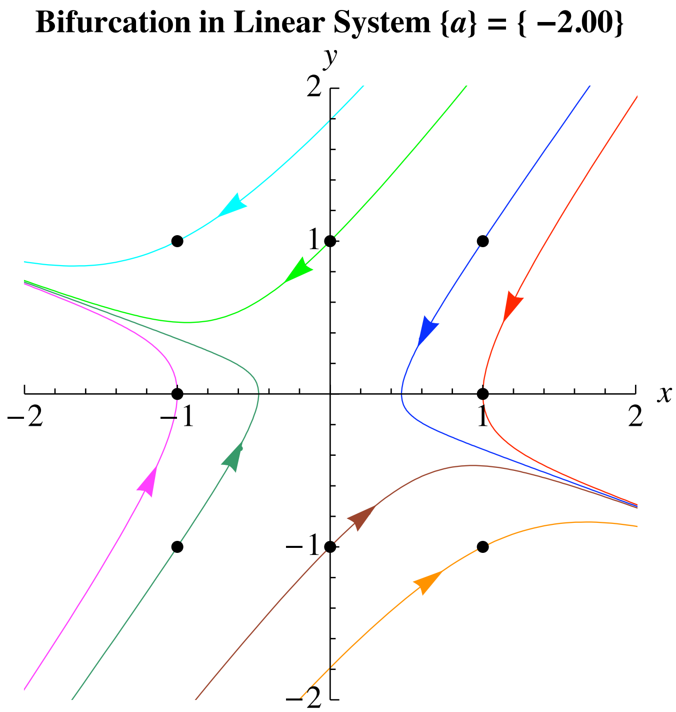
```
plrange = {{-2, 2}, {-2, 2}}; asprat = 1; imsize = 250;

arrowflag = True; arrowvec = {1 / 2};

t0 = 0.0; h = 0.02; nsteps = 800; bothdirflag = True;

rangeflag = True; ranger = {{-2.1, 2.1}, {-2.1, 2.1}};

initset =
  {{1, 0}, {1, 1}, {0, 1}, {-1, 1}, {-1, 0}, {-1, -1}, {0, -1}, {1, -1}};
```

Now we specify the list of a-values for which the portraits are to be constructed.

```
setcolor[{Black}];

ptsize = 0.02;

refgraph = dots[initset];

parmlist = Module[{ans, i}, ans = {};
  Do[ans = Append[ans, {i / 10.}], {i, -20, 20}]; ans]

{{-2.}, {-1.9}, {-1.8}, {-1.7}, {-1.6}, {-1.5}, {-1.4},
 {-1.3}, {-1.2}, {-1.1}, {-1.}, {-0.9}, {-0.8}, {-0.7}, {-0.6},
 {-0.5}, {-0.4}, {-0.3}, {-0.2}, {-0.1}, {0}, {0.1}, {0.2},
 {0.3}, {0.4}, {0.5}, {0.6}, {0.7}, {0.8}, {0.9}, {1.}, {1.1},
 {1.2}, {1.3}, {1.4}, {1.5}, {1.6}, {1.7}, {1.8}, {1.9}, {2.}}

setcolor[
  {Red, Blue, Green, Cyan, Magenta, SeaGreen, BurntUmber, Orange}];
```

Now we use the function bifurc to construct a long sequence suitable for a movie.

```
bifurc[initset, t0, h, nsteps, 1, 2, parmlist, refgraph]

Bifurcation sequence for parmlist =
 {{-2.}, {-1.9}, {-1.8}, {-1.7}, {-1.6}, {-1.5}, {-1.4}, {-1.3}, {-1.2}, {-1.1},
  {-1.}, {-0.9}, {-0.8}, {-0.7}, {-0.6}, {-0.5}, {-0.4}, {-0.3}, {-0.2},
  {-0.1}, {0}, {0.1}, {0.2}, {0.3}, {0.4}, {0.5}, {0.6}, {0.7}, {0.8}, {0.9},
  {1.}, {1.1}, {1.2}, {1.3}, {1.4}, {1.5}, {1.6}, {1.7}, {1.8}, {1.9}, {2.}}
```



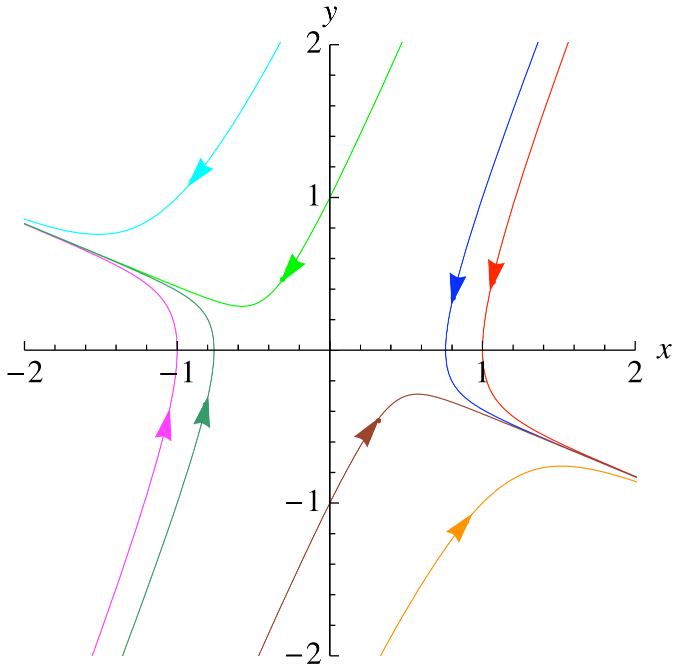
The movie sequence is too long to print out, so we construct a second, much shorter sequence with the following parameter values: a = -1 (saddle), a = 0 (saddle-node transition), a = 0.5 (stable node), a = 1 (spiral-node transition), and a = 2 (spiral).

```
parmlist = {{-1}, {0}, {0.5}, {1}, {2}};
```

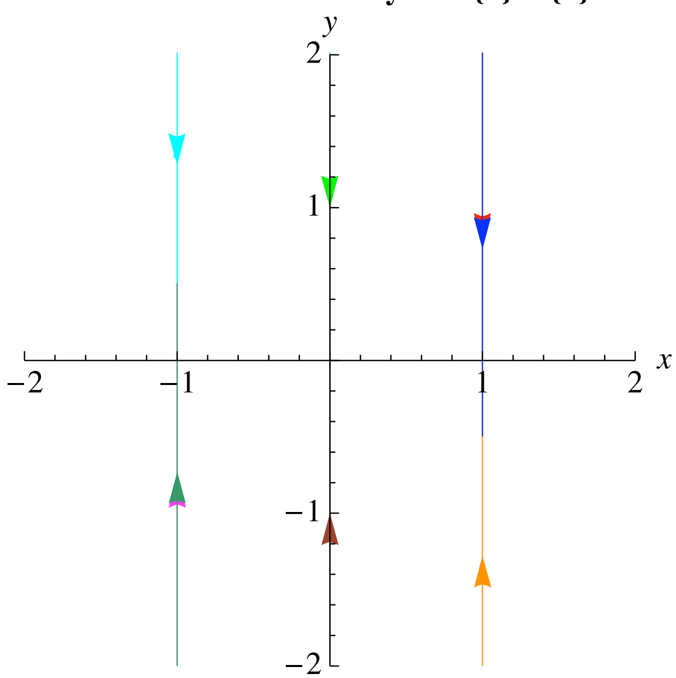
```
bifurc[initset, t0, h, nsteps, 1, 2, parmlist]
```

```
Bifurcation sequence for parmlist = {{-1}, {0}, {0.5}, {1}, {2}}
```

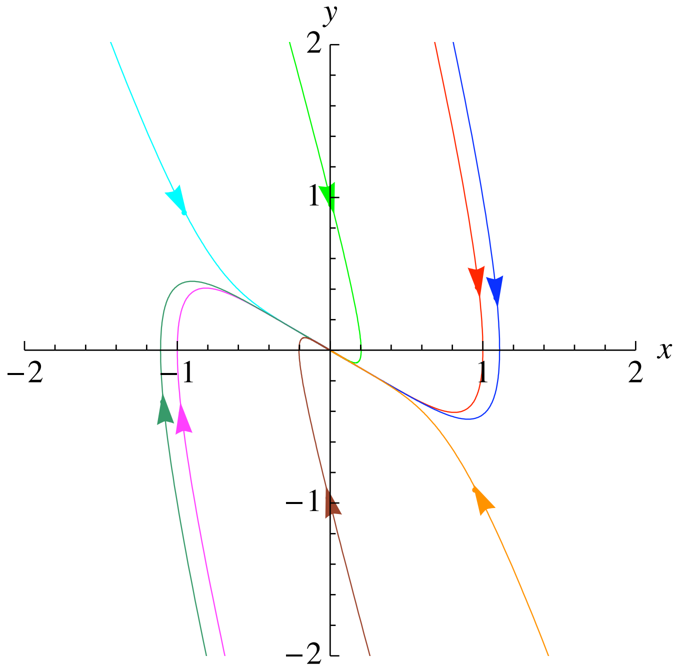
Bifurcation in Linear System {a} = {−1}



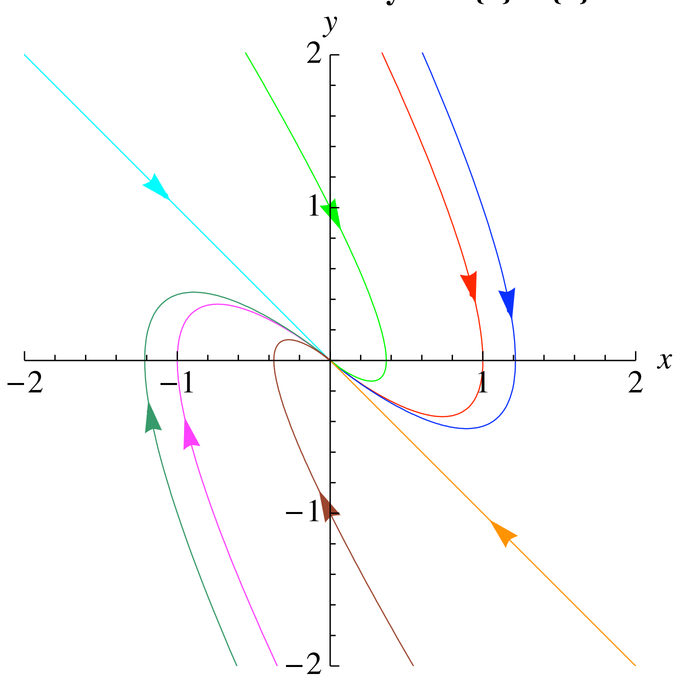
Bifurcation in Linear System {a} = {0}

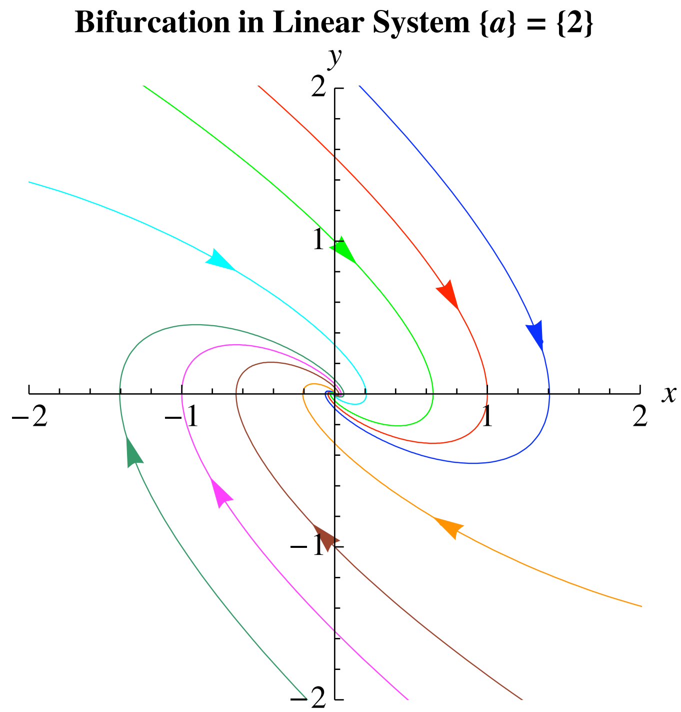


Bifurcation in Linear System {*a*} = { 0.50}



Bifurcation in Linear System {*a*} = {1}




Bifurcation in Linear System {a} = {2}
y
2
1
−1
−2
x
−2
−1
1
2