ME 406
Distortion of Areas in a 2D Flow

We consider the system of differential equations given by

$$\dot{x} = x\ ,\ \dot{y} = -2y\ , \tag{1}$$

with solution

$$x(t) = x(0)e^{t},\ y(t) = y(0)e^{-2t}\ . \tag{2}$$

The eigenvalues are 1 and -2, the divergence is -1 and the flow is dissipative. Areas will shrink according to the law

$$A(t) = A(0)e^{-t}\ . \tag{3}$$

In spite of the fact that areas shrink, neighboring solutions can become separated, because any initial separation in x will grow exponentially. To visualize this geometrically, we follow the transformation of a square of initial conditions. The initial vertices of the square are {1,1}, {-1,1}, {-1,-1}, and {1,-1}. The sequence of graphs below, which can be animated to form a movie, shows the transformation of this square by the flow.

First we define the four vertices of the distorted square.

```
ClearAll[p1, p2, p3, p4]
```

```
p1 = {Exp[t], Exp[-2 t]};
```

```
p2 = {-Exp[t], Exp[-2 t]};
```

```
p3 = {-Exp[t], -Exp[-2 t]};
```

```
p4 = {Exp[t], -Exp[-2 t]};
```

Now we define parametrically the four sides of the distorted square.

```
side1 = (1 - u) * p1 + u * p2;
```

```
side2 = (1 - u) * p2 + u * p3;
```

```
side3 = (1 - u) * p3 + u * p4;
```

```
side4 = (1 - u) * p4 + u * p1;
```

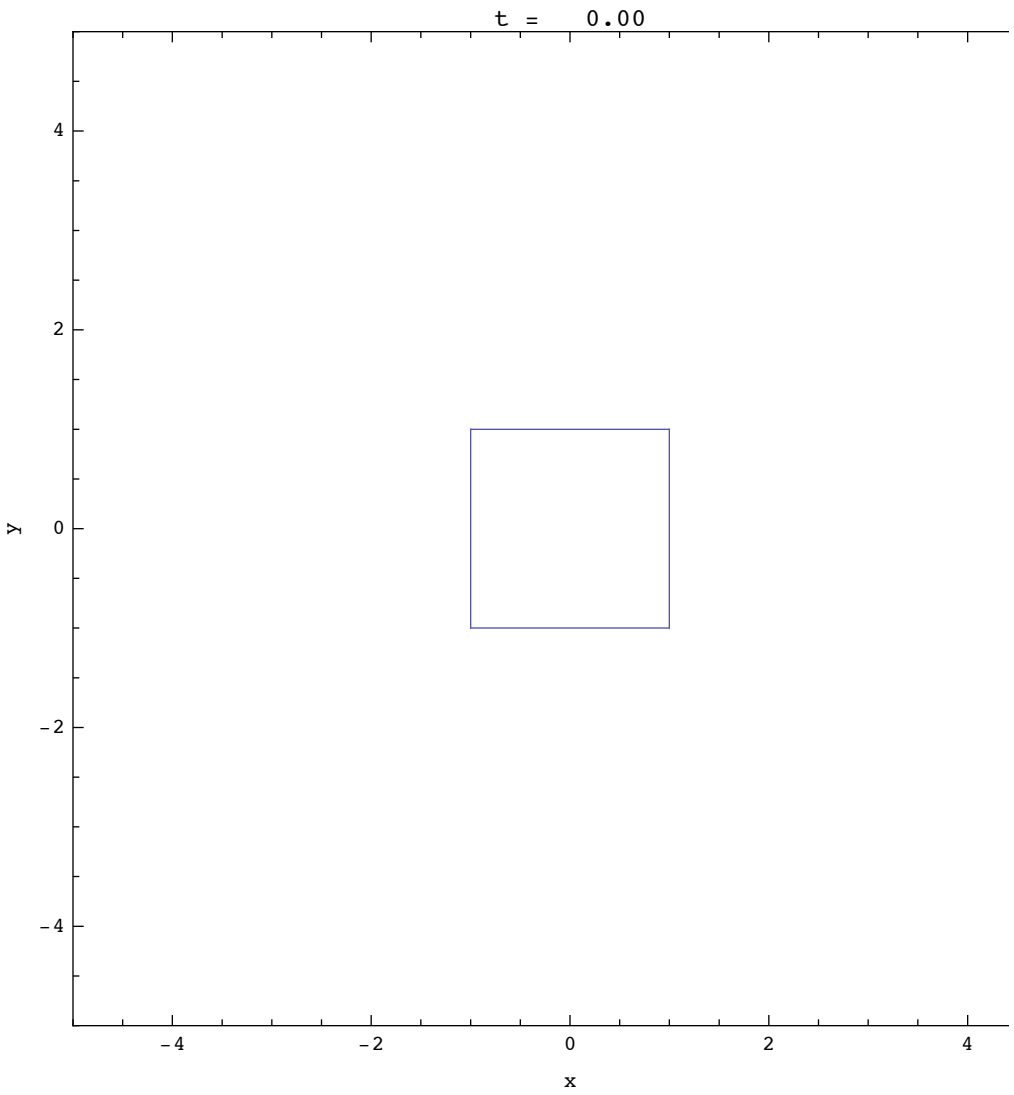
```
square = {side1, side2, side3, side4};
```

The function squareplot[t], defined below, produces a plot of the distorted square at time t.

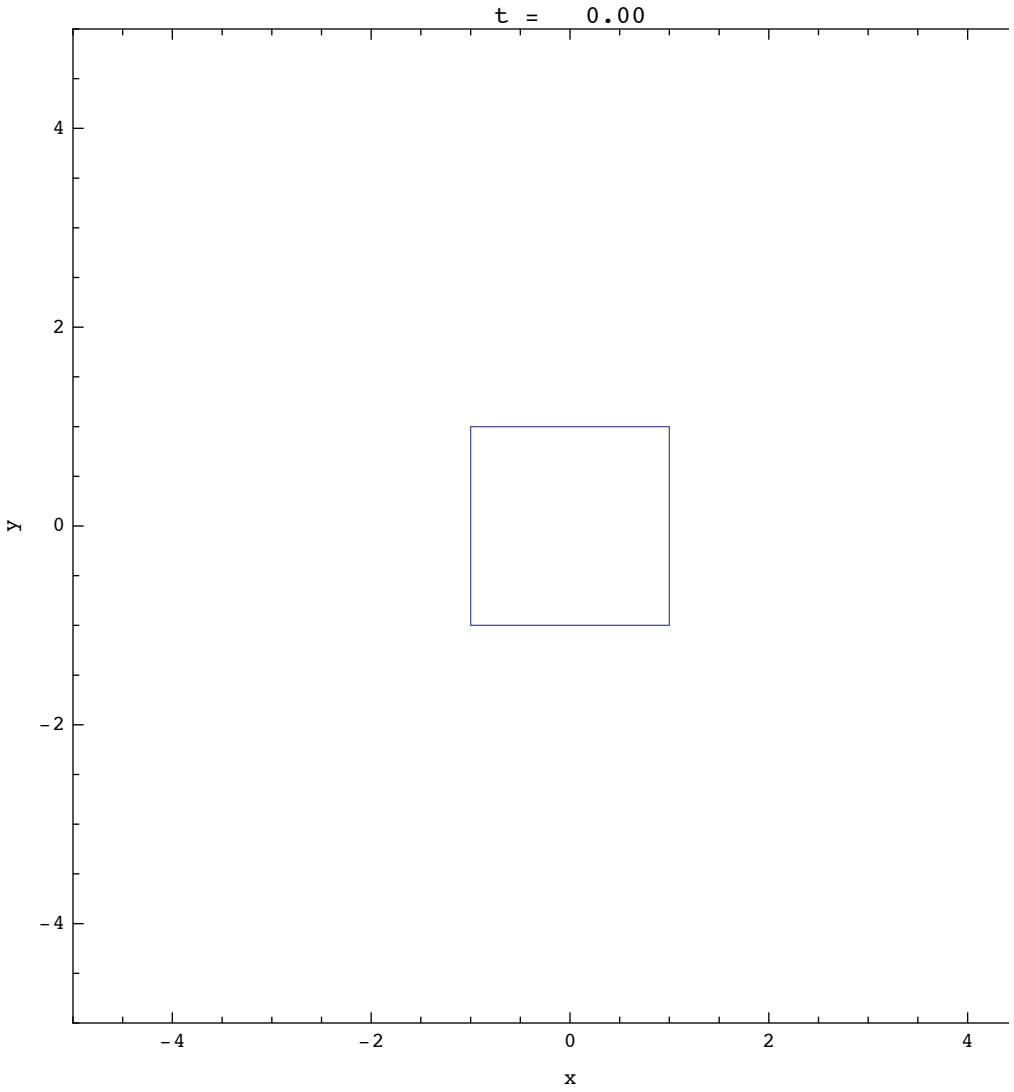
```
squareplot[time_] := Module[{plotter, temgraph, i}, temgraph = {};
  Do[plotter = N[square[[i]] /. t → time];
   temgraph = Append[temgraph, ParametricPlot[plotter, {u, 0, 1},
      PlotRange → {{-5, 5}, {-5, 5}}, FrameLabel → {"x", "y"}, Axes →
       False, PlotLabel → Row[{"t = ", PaddedForm[time, {4, 2}]}],
      AspectRatio → 1, ImageSize → 400,
      Frame → True]], {i, 1, 4}]; Print[Show[temgraph]]]
```

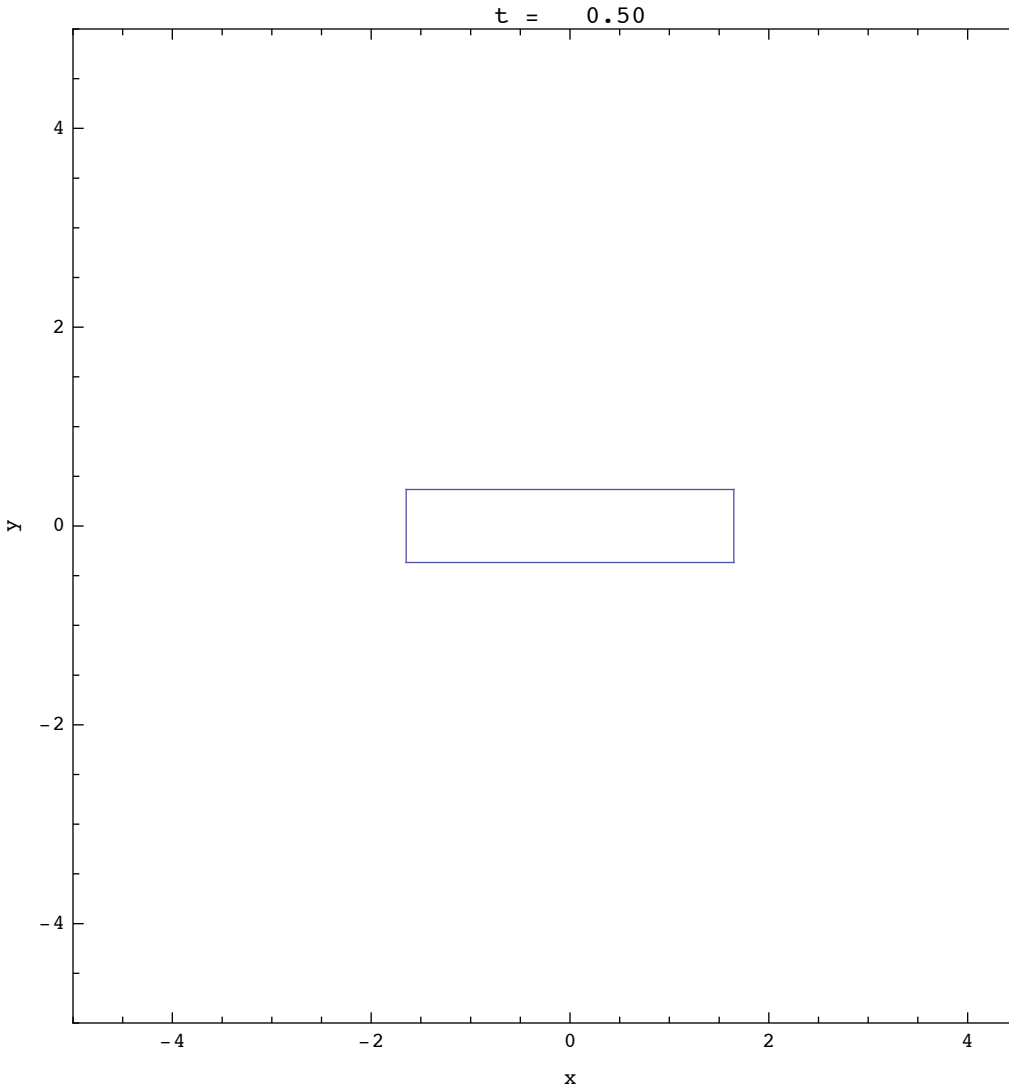
Now we use a Do loop to produce the graph sequence, which can then be animated as a movie. The printed version of the file shows only the first graph in the long sequence and then four representative graphs.

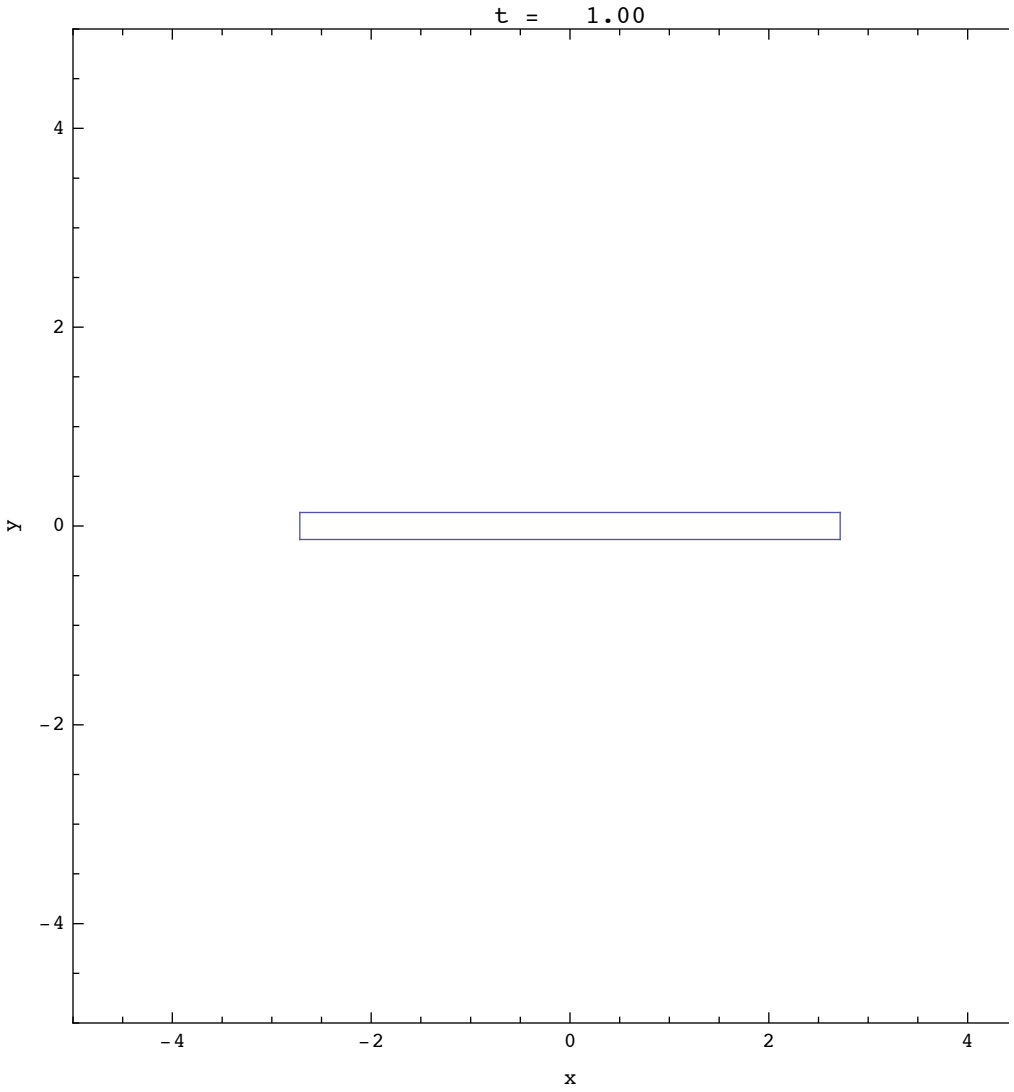
```
Do[squareplot[0.05 * i], {i, 0, 30}];
```

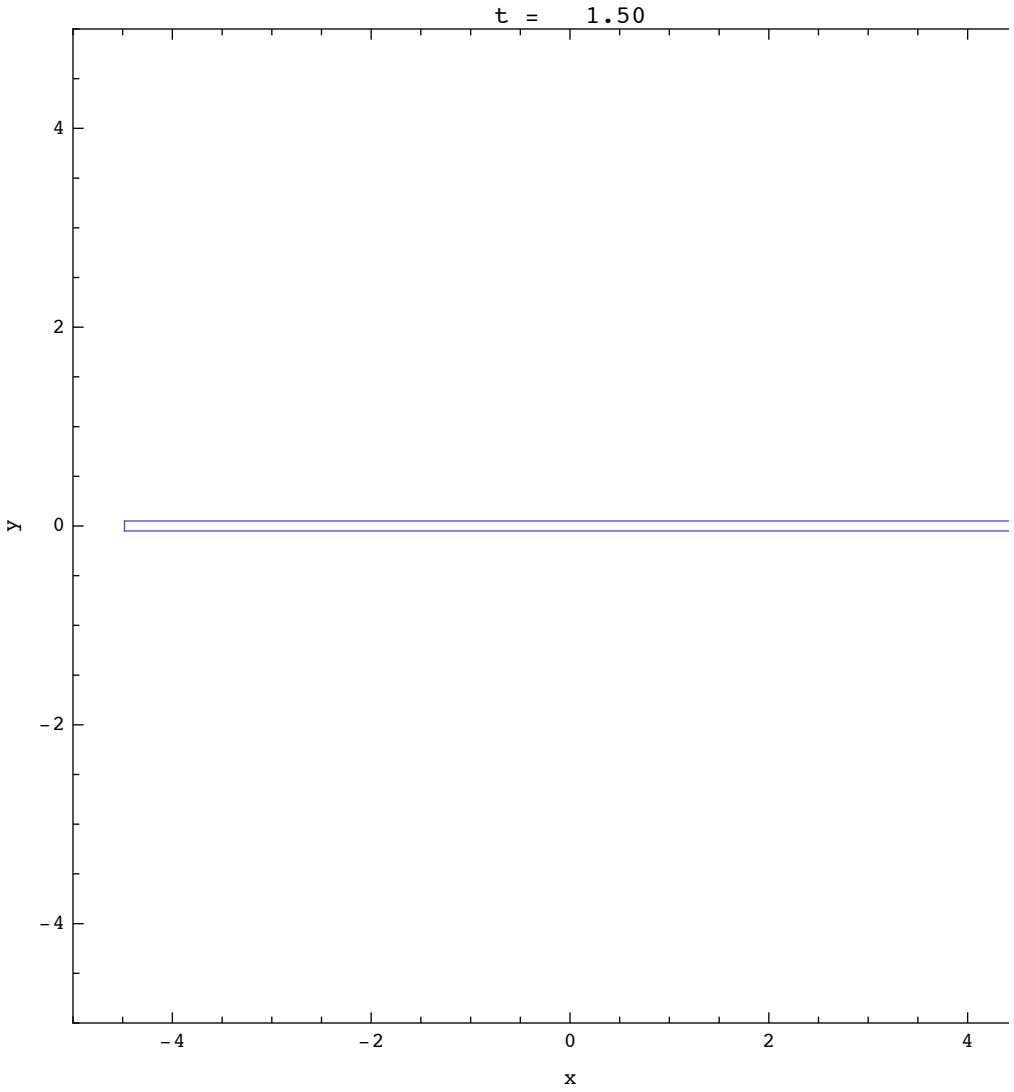



```
Do[squareplot[0.5 * i], {i, 0, 3}];
```


t = 0.00
4
2
0
−2
−4
y
−4
−2
0
2
4
x


t = 0.50
4
2
0
−2
−4
y
−4
−2
0
2
4
x


t = 1.00
4
2
0
−2
−4
y
−4
−2
0
2
4
x


t = 1.50
4
2
0
−2
−4
y
−4
−2
0
2
4
x