Lorenz Equations: An Introduction

```
In[441]:= sysid

        Mathematica 4.1, DynPac 10.65, 1/14/2002

In[442]:= plotreset; intreset;
```

In this notebook we use DynPac to take a preliminary look at solutions of the Lorenz equations. The primary point illustrated here is the sensitive dependence on initial conditions that is characteristic of chaotic solutions. The Lorenz equations are given below.

$$\frac{dx}{dt} = \sigma (y - x),$$

$$\frac{dy}{dt} = r x - y - x z,$$

$$\frac{dz}{dt} = x y - b z.$$

These equations contain three parameters: σ, r, and b. Now we specify this system for DynPac.

```
In[443]:= setstate[{x, y, z}];

In[444]:= setparm[{r, b, σ}];

In[445]:= slopevec = {σ * (y - x), r * x - y - x * z, x * y - b * z};
```

We name the system Lorenz.

```
In[446]:= sysname = "Lorenz";
```

Now we specify a particular set of parameter values:

```
In[447]:= parmval = {28.0, 8 / 3, 10.0};
```

Before carrying out the integration, we must specify the initial vector, for which we use two values called initialvec1 and initialvec2, and the time step which we call h, the number of time steps which we call nsteps, and the initial time t0.

```
In[448]:= initialvec1 = {0.0, 1.0, 0.0};

In[449]:= initialvec2 = {0.0, 1.001, 0.0};

In[450]:= h = 0.015;

In[451]:= nsteps = 2000;
```

```
In[452]:= t0 = 0.0;
```

Now we integrate the equations for each initial vector, and name the solutions lorenz1 and lorenz2.

```
In[453]:= lorenz1 = integrate[initialvec1, t0, h, nsteps];
```

```
In[454]:= lorenz2 = integrate[initialvec2, t0, h, nsteps];
```

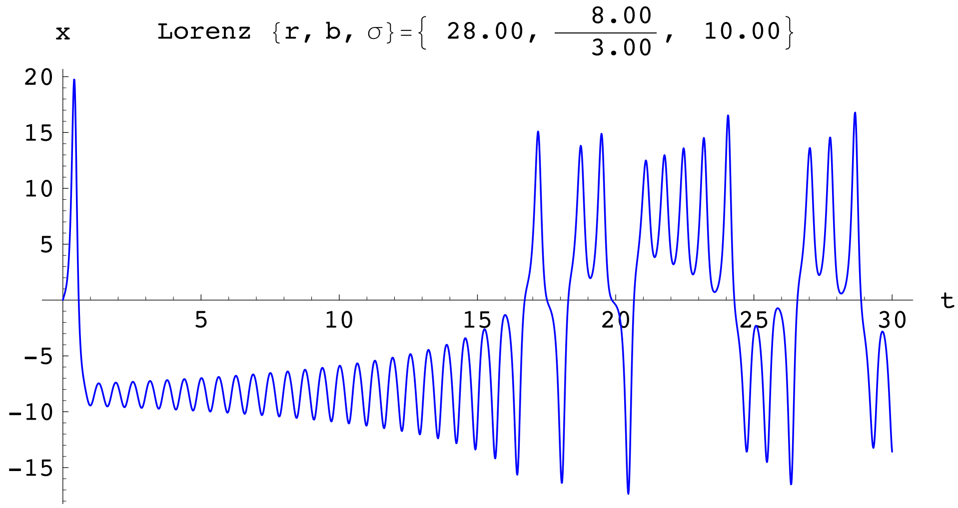
We plot the x component versus time for each solution, using an aspect ratio of 0.5 for the graph.

```
In[455]:= asprat = 0.5;
```

```
In[456]:= imsize = 380;
```

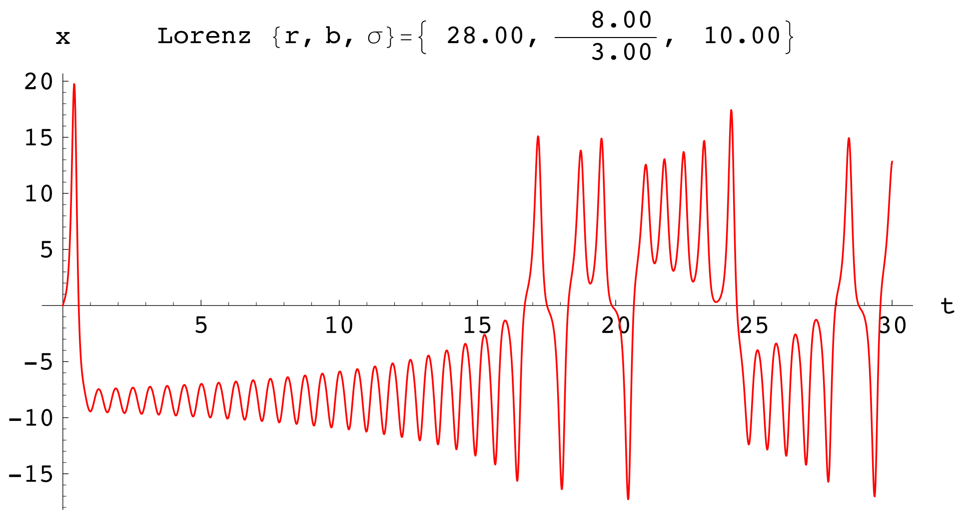
```
In[457]:= setcolor[{Blue}];
```

```
In[458]:= lorplot1 = timeplot[lorenz1, 1];
```



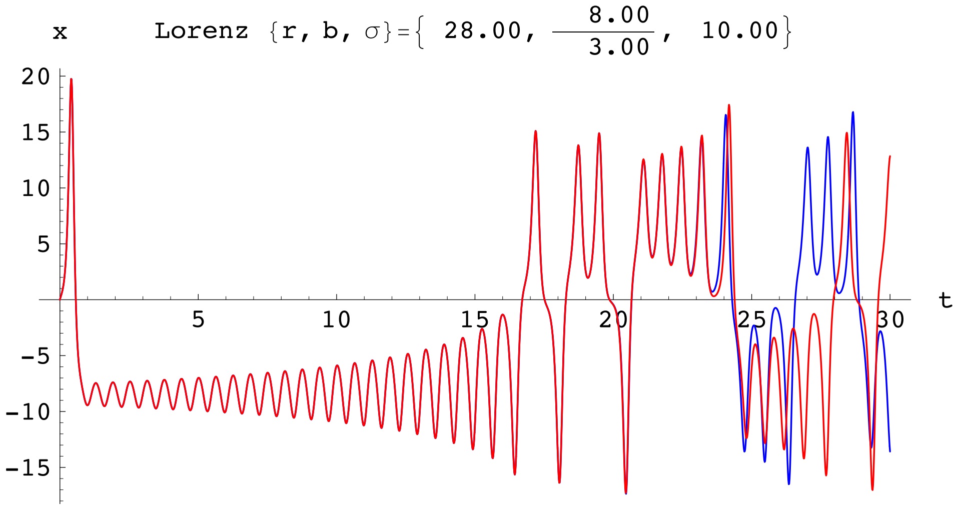

```
In[459]:= setcolor[{Red}];
```

```
In[460]:= lorplot2 = timeplot[lorenz2, 1];
```



These two solutions have initial conditions which differ only by 0.001 out of 1 in the y-coordinate. Let's plot them on the same graph.

```
In[461]:= show[lorplot1, lorplot2];
```



We see that the solutions eventually diverge completely, in spite of their very small difference initially. This is known as sensitive dependence on initial conditions.