

# ME 406

## Bifurcations I

### Bifurcations in a Simple Linear System

`sysid`

Mathematica 4.1.2, DynPac 10.66, 3/5/2002

`intreset; plotreset;`

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## ■ 1. Introduction

In this series of notebooks on bifurcations, we look at simple examples of common bifurcations. We follow in part the discussion given in Chapter 3 of **Nonlinear Dynamics and Chaos**, Steven Strogatz, Addison-Wesley, 1994. The examples in all of the notebooks include the following: saddle-node bifurcation, transcritical bifurcation, supercritical pitchfork bifurcation, subcritical pitchfork bifurcation, and Hopf bifurcation. In this first notebook, we look at a very simple linear system with a single parameters. As the parameter is changed, the equilibrium goes through the following sequence: saddle, stable node, stable spiral. We construct a movie of the change with the parameter of the system orbits.

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## ■ 2. Definition of the System

We consider the following system, depending on one parameter  $a$ :

$$\dot{x} = ay, \quad \dot{y} = -x - 2y.$$

This system in general has a single equilibrium point at (0,0), the character of which depends on  $a$ . We begin our analysis by defining the system for DynPac

```
setstate[{x, y}]; setparm[{a}]; slopevec = {a y, -x - 2 y};  
sysname = "Bifurcation in Linear System";
```

We give a name to the equilibrium point at the origin.

```
eq = {0, 0};
```

We also note that when  $a = 0$ , the system is degenerate, and the entire line  $y = -x/2$  is an equilibrium line. This suggests that  $a = 0$  is an important value in the bifurcation study. We get the eigenvalues at the equilibrium.

```
eigsys[eq]
```

```
{{-1 - sqrt[1 - a], -1 + sqrt[1 - a]}, {{-1 + sqrt[1 - a], 1}, {-1 - sqrt[1 - a], 1}}}
```

From the expressions for the eigenvalues, we conclude the following: for  $a < 0$ , the equilibrium is a saddle; for  $0 < a < 1$ , the equilibrium is a stable node; for  $1 < a$ , the equilibrium is a stable spiral. Thus the important bifurcations are at  $a = 0$  (transition between saddle and node), and  $a = 1$  (transition between node and spiral). The most important bifurcation is at  $a = 0$  because it is a stability transition. We now construct a sequence of phase plane plots showing the bifurcations. Each plot shows a representative set of integral curves for a given value of  $a$ . The connection between the plots is that the integral curves are always anchored at the same spots on the edges of the plotting window, and a specific curve color is associated with each anchor spot. On the graphs, we place a black dot at each of the anchor points. We set the parameters for integrating and plotting.

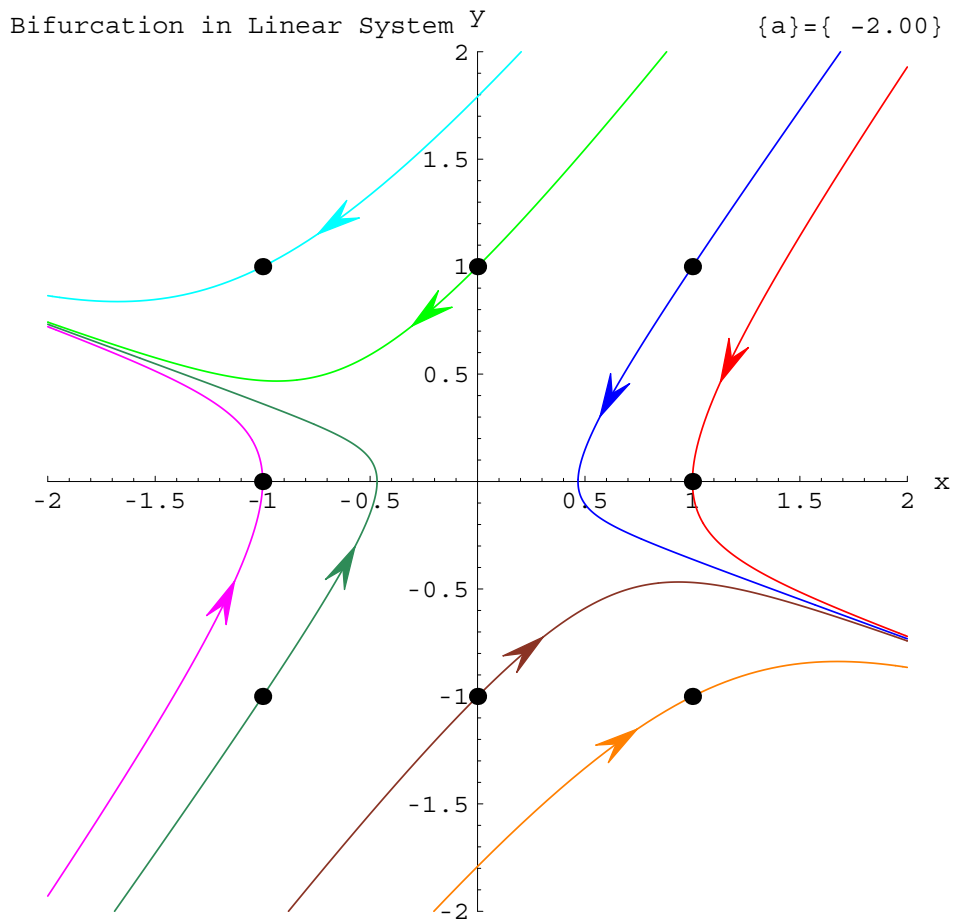
```
plrange = {{-2, 2}, {-2, 2}}; asprat = 1; labshift = 18; imsize = 360;
arrowflag = True; arrowvec = {1/2};
t0 = 0.0; h = 0.02; nsteps = 800; bothdirflag = True;
rangeflag = True; ranger = {{-2.1, 2.1}, {-2.1, 2.1}};
initset = {{1, 0}, {1, 1}, {0, 1},
           {-1, 1}, {-1, 0}, {-1, -1}, {0, -1}, {1, -1}};
```

Now we specify the list of  $a$ -values for which the portraits are to be constructed.

```
setcolor[{Black}];
display = False;
ptsize = 0.02;
refgraph = dots[initset];
display = True;
parmlist = Module[{ans, i}, ans = {};
  Do[ans = Append[ans, {i/10.}], {i, -20, 20}]; ans]
{{-2.}, {-1.9}, {-1.8}, {-1.7}, {-1.6}, {-1.5}, {-1.4}, {-1.3},
{-1.2}, {-1.1}, {-1.}, {-0.9}, {-0.8}, {-0.7}, {-0.6},
{-0.5}, {-0.4}, {-0.3}, {-0.2}, {-0.1}, {0}, {0.1}, {0.2},
{0.3}, {0.4}, {0.5}, {0.6}, {0.7}, {0.8}, {0.9}, {1.}, {1.1},
{1.2}, {1.3}, {1.4}, {1.5}, {1.6}, {1.7}, {1.8}, {1.9}, {2.}}
setcolor[
{Red, Blue, Green, Cyan, Magenta, SeaGreen, BurntUmber, Orange}];
```

Now we use the function `bifurc` to construct a long sequence suitable for a movie.

```
bifurc[initset, t0, h, nsteps, 1, 2, parmlist, refgraph];
Bifurcation sequence for parmlist =
{{-2.}, {-1.9}, {-1.8}, {-1.7}, {-1.6}, {-1.5}, {-1.4}, {-1.3}, {-1.2}, {-1.1},
{-1.}, {-0.9}, {-0.8}, {-0.7}, {-0.6}, {-0.5}, {-0.4}, {-0.3}, {-0.2},
{-0.1}, {0}, {0.1}, {0.2}, {0.3}, {0.4}, {0.5}, {0.6}, {0.7}, {0.8}, {0.9},
{1.}, {1.1}, {1.2}, {1.3}, {1.4}, {1.5}, {1.6}, {1.7}, {1.8}, {1.9}, {2.}}
```



The movie sequence is too long to print out, so we construct a second, much shorter sequence with the following parameter values:  $a = -1$  (saddle),  $a = 0$  (saddle-node transition),  $a = 0.5$  (stable node),  $a = 1$  (spiral-node transition), and  $a = 2$  (spiral).

```
parmlist = {{-1}, {0}, {0.5}, {1}, {2}};
```

```
bifurc[initset, t0, h, nsteps, 1, 2, parmlist];
```

```
Bifurcation sequence for parmlist = {{-1}, {0}, {0.5}, {1}, {2}}
```

