

ME 406

Distortion of Areas in a 2D Flow

We consider the system of differential equations given by

$$\dot{x} = x, \quad \dot{y} = -2y, \quad (1)$$

with solution

$$x(t) = x(0)e^t, \quad y(t) = y(0)e^{-2t}. \quad (2)$$

The eigenvalues are 1 and -2, the divergence is -1 and the flow is dissipative. Areas will shrink according to the law

$$A(t) = A(0)e^{-t}. \quad (3)$$

In spite of the fact that areas shrink, neighboring solutions can become separated, because any initial separation in x will grow exponentially. To visualize this geometrically, we follow the transformation of a square of initial conditions. The initial vertices of the square are $\{1,1\}$, $\{-1,1\}$, $\{-1,-1\}$, and $\{1,-1\}$. The sequence of graphs below, which can be animated to form a movie, shows the transformation of this square by the flow.

First we define the four vertices of the distorted square.

```
ClearAll [p1, p2, p3, p4]
p1 = {Exp[t], Exp[-2 t]};
p2 = {-Exp[t], Exp[-2 t]};
p3 = {-Exp[t], -Exp[-2 t]};
p4 = {Exp[t], -Exp[-2 t]};
```

Now we define parametrically the four sides of the distorted square.

```
side1 = (1 - u) * p1 + u * p2;
side2 = (1 - u) * p2 + u * p3;
side3 = (1 - u) * p3 + u * p4;
side4 = (1 - u) * p4 + u * p1;
square = {side1, side2, side3, side4};
```

The function `squareplot[t]`, defined below, produces a plot of the distorted square at time t .

```

squareplot[time_] := Module[{plotter, temgraph, i}, temgraph = {};
  Do[plotter = N[square[[i]] /. t → time];
  temgraph = Append[temgraph, ParametricPlot[plotter, {u, 0, 1},
    PlotRange → {{-5, 5}, {-5, 5}}, FrameLabel → {"x", "y"}, Axes →
    False, PlotLabel → SequenceForm["", "t = ",
    PaddedForm[time, {4, 2}]], AspectRatio → 1, ImageSize → 400,
    Frame → True, DisplayFunction → Identity]], {i, 1, 4}];
  Show[temgraph, DisplayFunction → $DisplayFunction]]

```

The command below is necessary to turn off a repeated and annoying message from *Mathematica*.

```
Off[ParametricPlot::ppcom]
```

Now we use a Do loop to produce the graph sequence, which can then be animated as a movie. The printed version of the file shows only four of the graphs.

```
Do[squareplot[0.05 * i], {i, 0, 30}];
```

