Problem 1

1) Steam expands isentropically through a converging nozzle from a large tank at 10 MPa, 360°C. The exit area of the nozzle is 5 cm². If the nozzle is choked at an exit pressure of 5.5 MPa, determine the exit velocity in m/s, the exit plane pressure in bar, and the mass flow rate of steam in kg/s, for back pressures of (a) 8 MPa, (b) 6 MPa, (c) 4 MPa.

\[ P_1 = 10 \text{ MPa} \]
\[ T_1 = 360°C \]

(a) \( P_0 = 8 \text{ MPa} = P_2 \)

\[ S_2 = S_1 = 6.006 \text{ J/kg K} \]

\[ dh = -v dV \]

\[ h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2} \]

\[ V_1 = 0 \text{ large tank} \]

\[ V_2 = \sqrt{2(h_1-h_2)} \]

\[ \frac{V_2}{V_1} = \frac{2962.1 \text{ m/s}}{6.1872 \text{ m/s}} = \frac{5.9487 - 6.1872}{5.9487 - 6.1872} \rightarrow h_2 = 2912.18 \text{ J/kg} \]

\[ V_2 = \sqrt{2(2962.1 - 2912.18)^2} = \sqrt{99.907 \text{ m/s}} \]

\[ V_2 = \frac{\sqrt{99.907 \text{ m/s}}}{5 \text{ m/s}} \rightarrow V_2 = 316.08 \text{ m/s} \]

\[ m = \frac{A_2 V_2}{V_1} = \left( 5 \text{ cm}^2 \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 \left( 816.08 \text{ m/s} \right) \]

\[ \frac{0.027817 \text{ m}^3}{\text{ kg}} \rightarrow m = 5.68 \text{ kg/s} \]

\[ V_2 = \frac{0.027817 - 0.03087}{5.9487 - 6.1872} \rightarrow V_2 = 0.027817 \text{ m}^3 \]
b) \( P_1 = P_6 = 6 \text{ MPa} \)

\[
\frac{\Delta h}{\Delta h} = \frac{2957.6}{6.0660 - 6.1846} \quad \Rightarrow \quad \Delta h = 2850.42 \frac{\text{kJ}}{\text{kg}}
\]

\[
\frac{\Delta V}{\Delta h} = \frac{0.028876}{6.0660 - 6.1846} \quad \Rightarrow \quad \Delta V = 0.08491 \frac{\text{m}^3}{\text{kg}}
\]

\[
V_2 = \sqrt{2 \left( 2962.1 - 2850.42 \right) \times 10^3 \frac{\text{m}^3}{s^2}} \quad \Rightarrow \quad V_2 = 472.603 \frac{\text{m}}{\text{s}}
\]

\[
\dot{m} = \frac{A_2 V_2}{\frac{\Delta V}{\Delta h}} = \frac{5 \text{ cm}^2}{(100 \text{ cm})^2} \frac{472.603}{0.08491} \quad \Rightarrow \quad \dot{m} = 6.26 \text{ kg/s}
\]

(c) \( P = 4 \text{ MPa} \)

\[
R = 60 \text{ bar} \quad h_2 = 2827.4 \frac{\text{kJ}}{\text{kg}} \quad V_2 = 0.00387 \frac{\text{m}^3}{\text{kg}}
\]

\[
V_1 = \sqrt{2 \left( 2962.1 - 2827.4 \right) \times 10^3} \quad \Rightarrow \quad V_2 = 519 \text{ m/s}
\]

\[
\dot{m} = \frac{A_2 V_2}{V_2} = \frac{519}{(100 \text{ cm})^2 \left( 0.03491 \right)} = 6.7 \text{ kg/s}
\]
PROBLEM # 2

**KNOWN:** A converging-diverging nozzle operates at steady state with isentropic flow of air.

**FIND:** Determine specified conditions at the nozzle exit and throat.

**SCHEMATIC & GIVEN DATA:**

1. \( M_1 = 0.2 \)
2. \( M_e = 1 \)
3. \( P_1 = 8 \text{ bar} \)
4. \( T_1 = 400 \text{ K} \)
5. \( A_1 = 3 \text{ cm}^2 \)
6. \( A_2 = 6 \text{ cm}^2 \)

**ASSUMPTION:** The air behaves as an ideal gas, with \( k = 1.4 \).

**ANALYSIS:** Use Table 9.1.

First, find the stagnation state.

\[
M_1 = 0.2 \Rightarrow T_1 / T_0 = 0.99206 \Rightarrow T_0 = 403.2 \text{ K}
\]

\[
P_1 / P_0 = 0.97250 \Rightarrow P_0 = 8.226 \text{ bar}
\]

If the nozzle flow is choked, \( M_e = 1 \), and

\[
T_e = T_0 (1.83333) = 336.0 \text{ K}
\]

\[
P_e = P_0 (.52828) = 4.3456 \text{ bar}
\]

Further,

\[
V_{th} = \frac{RT_e}{P_e} = 0.2219 \text{ m}^3/\text{kg}
\]

\[
V_{th} = (1) \sqrt{\frac{k R T_{th}}{M_1}} = 367.4 \text{ m/s}
\]

Thus, the mass flow rate is

\[
\dot{m} = \frac{A_1 \sqrt{V_e}}{V_{th}} = 0.497 \text{ kg/s}
\]

Now, with \( A_2 / A_1 = 6 \text{ cm}^2 / 3 \text{ cm}^2 = 2 \), there are two cases in Table 9.1.

**Supersonic**

\[
M_2 > 2.20 \Rightarrow P_2 = (.09352) P_0 = 0.769 \text{ bar} \Rightarrow \text{P}_2
\]

\[
T_2 = (.50813) T_0 = 204.9 \text{ K} \Rightarrow \text{T}_2
\]

**Subsonic**

\[
M_1 < 0.3079 \Rightarrow P_2 = (.93601) P_0 = 7.70 \text{ bar} \Rightarrow \text{P}_2
\]

\[
T_2 = (.98127) T_0 = 395.6 \text{ K} \Rightarrow \text{T}_2
\]
PROBLEM # 3

**KNOWN:** Air expands isentropically through a converging nozzle and discharges to the atmosphere. The inlet conditions are specified and the exit plane area is given.

**FIND:** Determine the mass flow rate. Is it possible to increase the mass flow rate by increasing the supply region pressure?

**SCHEMATIC & GIVEN DATA:**

![Diagram](image)

**ASSUMPTIONS:** (1) The control volume is at steady state, with \( \dot{V} = 0 \).
(2) The air expands isentropically.
(3) The air behaves as an ideal gas.
(4) Because of the small pressure difference, the temperature range will be limited. Therefore, assume \( k = 1.4 \) is constant.

**ANALYSIS:** (a) First, determine if the nozzle is choked. Using Eq. 9.51 with \( M = 1 \) gives

\[
\frac{p_0}{p_s} = \left( \frac{k+1}{2} \right)^{\frac{k-1}{k}} = 1.8929 \Rightarrow p_s = 0.7396 \text{ bar}.
\]

\( \text{Not choked} \)

so \( p_s = 1 \text{ bar} \)

For the isentropic expansion

\[
T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{k-1}{k}} = 254.3 \text{ K}
\]

The exit velocity is, with \( q = 1.004 \text{ kJ/kg.k} \),

\[
V_2 = \sqrt{2q(T_1 - T_e)}
\]

\[
= \sqrt{2(1.004 \text{ kJ/kg.k})(280-254.3) \text{K}} \left( \frac{10^3 \text{ N.m}}{1 \text{ kJ}} \right) \left( \frac{1 \text{ kg.m/s}^2}{1 \text{ N} \cdot \text{m}} \right)
\]

\[
= 227.2 \text{ m/s}.
\]

Thus, the mass flow rate is

\[
m = \frac{A_2 V_2 p_2}{RT_2} = \left( 0.0013 \text{ m}^2 \right) \left( 227.2 \text{ m/s} \right) \left( 1 \text{ bar} \right) \left( \frac{8.314 \text{ kJ}}{287.1 \text{ kJ/kg.K}} \right) \left( 254.3 \text{ K} \right)
\]

\[
= 0.4047 \text{ kg/s}.
\]

(b) Next, consider the case for which \( p_0 = 2 \text{ bar}, T_0 = 280K \) and \( p_s = 1 \text{ bar} \).

With Eq. 9.51

\[
\frac{p_0}{p_m} = \left( \frac{k+1}{2} \right)^{\frac{k-1}{k}} = 1.8929 \Rightarrow p_m = 1.0569 \text{ bar}.
\]

\( p_s < p_m \Rightarrow \text{choked conditions} \Rightarrow p_2 = p_m = 1.0569 \text{ bar} \).
Thus, \( T_2 = T^* \). From Eq. 9.50 with \( M = 1 \):
\[
\frac{T_0}{T_{in}} = \frac{k+1}{2} = 1.2 \Rightarrow T^* = 233.33 \text{ K}
\]
The exit velocity is
\[
V_2 = \sqrt{\frac{2(1.004)(280-233.33)}{8.914} \left( \frac{233.33}{28.97} \right)} = 306.1 \text{ m/s}
\]
Thus, the mass flow rate is
\[
\dot{m} = \frac{(0.0013)(306.1)(1)}{(8.914/28.97)(233.33)} \mid \text{10}^3 \mid = 0.8943 \text{ kg/s}
\]

**PROBLEM 4**

**SCHEMATIC & GIVEN DATA:**

- \( P_i = 100 \text{ inHg} \)
- \( V_i = 0 \)
- \( T_i = 860 \text{°R} \)
- \( m = 4 \text{ lbs} \)
- \( P_2 \)
- \( 100 \text{ lbtsf} \)
- \( 860 \text{°R} \)

**ASSUMPTIONS:** (1) The control volume is at steady state, with \( Q_v = \dot{W}_v = 0 \).
(2) The air expands isentropically.
(3) The air behaves as an ideal gas.

**ANALYSIS:** For each case to be considered, \( P_i = 100 \text{ inHg} \), \( T_i = 860 \text{°R} \). Thus, \( h_i = 206.46 \text{ Btu/lb} \), \( P_i = 7.144 \). With \( P_2 \) specified

\[
\frac{P_2}{P_i} = \frac{P_2}{P_i} \Rightarrow h_2, T_2
\]

Then, the velocity is found using an energy balance

\[
V_2 = \sqrt{\frac{2(h_1 - h_2)}{k}}
\]

And the Mach number is

\[
M_2 = \frac{V_2}{\sqrt{kRT_2}}
\]

Finally, area \( A_2 \) is

\[
A_2 = \frac{\dot{m} V_2}{V_2 P_2} = \frac{\dot{m} R T_2}{V_2 P_2}
\]

(a) \( P_2 = (80/100)(7.144) = 5.7142 \Rightarrow h_2 = 193.67 \text{ Btu/lb} \), \( T_2 = 807.6 \text{°R} \)

\[
V_2 = \sqrt{\frac{2(206.46 - 193.67)}{1.4}} \left( \frac{778 \text{ ft} \cdot \text{lb}}{18 \text{ lb}} \right) \left( \frac{122 \text{ lb} \cdot \text{hr} \cdot \text{ft}^2}{1 \text{ hr} \cdot \text{ft}^2} \right) = 800.5 \text{ ft/s}
\]

From Table A-20, \( k = 1.386 \). Thus

\[
M_2 = \frac{\sqrt{(1.386)(1545 \text{ ft} \cdot \text{lb}) (8076 \text{°R}) (122 \text{ lb} \cdot \text{hr} \cdot \text{ft}^2)}}{(800.5 \text{ ft/s}) (80 \text{ lbf/in}^2)} = 0.577
\]

\[
A_2 = \frac{(4 \text{ lb/s}) (1545 \text{ ft} \cdot \text{lb}) (8076 \text{°R})}{(800.5 \text{ ft/s}) (80 \text{ lbf/in}^2)} = 1.848 \text{ ft}^2
\]