ME 251 - PROBLEM SET 4 – SOLUTIONS

PROBLEM 1

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: Same as Example 8.5, except that the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 120$ bar, $T_1 = 520^\circ$C $\Rightarrow h_1 = 3401.8 \text{ kJ/kg}, s_1 = 6.5555 \text{ kJ/kg.K}$

State 2: $p_2 = 10$ bar, $s_2 = s_1 \Rightarrow x_2 = 0.9931, h_2 = 2764.2 \text{ kJ/kg}$

State 3: $p_3 = 0.06 \text{ bar}, s_3 = s_1 \Rightarrow x_3 = 0.7727, h_3 = 2018.3 \text{ kJ/kg}$

State 4: $p_4 = 0.06 \text{ bar}, \text{ sat. liquid} \Rightarrow h_4 = 151.53 \text{ kJ/kg}$

State 5: $h_5 \approx h_4 + v_4 (P_5 - P_4)$

$= 151.53 + (1.0064 \times 10^{-3} \text{ m}^3/\text{kg})(10 - 0.06) \text{ bar} = 152.53 \text{ kJ/kg}$

State 6: $p_6 = 10 \text{ bar}, \text{ sat. liquid} \Rightarrow h_6 = 762.81 \text{ kJ/kg}$

State 7: $p_7 = 120 \text{ bar}, \Rightarrow h_7 = h_6 + v_6 (P_7 - P_6)$

$= 762.81 + (1.1273 \times 10^{-3} \text{ m}^3/\text{kg})(120 - 10) \text{ bar} = 775.21 \text{ kJ/kg}$

(a) Applying energy and mass balances to the control volume enclosing the open heater, the fraction of flow $y$ extracted at location 2 is

$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.81 - 152.53}{2764.2 - 152.53} = 0.2337$ \hspace{1cm} (1)

For the control volume surrounding the turbine stages

$\dot{W}_t = (h_1 - h_2) + (1 - y) (h_2 - h_3)$ \hspace{1cm} (2)

And, for the pumps

$\dot{W}_p = (h_7 - h_6) + (1 - y) (h_6 - h_4)$ \hspace{1cm} (3)
For the working fluid passing through the steam generator
\[ \dot{Q}_{in}/\dot{m}_1 = h_1 - h_7 = 3401.8 - 775.21 = 2626.6 \text{ kJ/kg} \] (4)
Thus, the thermal efficiency is
\[ \eta = \frac{\dot{W}_{el}/\dot{m}_1 - \dot{W}_f/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.455 \ (45.5\%) \Rightarrow \eta \] (5)
(b) The net power developed is
\[ \dot{W}_{net} = \dot{m}_1 \left( \dot{W}_{el}/\dot{m}_1 - \dot{W}_f/\dot{m}_1 \right) \] (7)
\[ \Rightarrow \dot{m}_1 = \frac{\dot{W}_{net}}{\left( \dot{W}_{el}/\dot{m}_1 - \dot{W}_f/\dot{m}_1 \right)} \]
\[ = \frac{(330 \times 10^3 \text{ kJ/s})}{(1209.2 - 13.17) \text{ kJ/kg}} \times \frac{3600}{1 \text{ W}} \]
\[ = 9.43 \times 10^5 \text{ kg/h} \]
PROBLEM 2

State 1: $P_1 = 80 \text{bar}$, $T_1 = 480^\circ \text{C}$. \(\Rightarrow h_1 = 33.46 \text{kJ/kg}, \quad \frac{51}{k^2} = 6.6586 \frac{\text{kJ}}{\text{kg}^2} \)

State 2: $P_2 = 20 \text{bar}$, $S_2 = 5\text{ bar}$. \(\Rightarrow h_2 = 2963.5 \frac{\text{kJ}}{\text{kg}} \)

State 3: $P_3 = 36\text{ bar}$, $S_3 = 5\text{ bar}$. \(\Rightarrow h_3 = 1.6718 \frac{\text{kJ}}{\text{kg}}, \quad S_3 = 8.9917 \frac{\text{kJ}}{\text{kg}}, \quad \frac{1}{h_4} = 0.9756 \frac{\text{kJ}}{\text{kg}} \)

State 4: $P_4 = 0.08 \text{ bar}$, $S_4 = 5\text{ bar}$. \(\Rightarrow h_4 = 5.2842 \frac{\text{kJ}}{\text{kg}} \)

State 5: $P_5 = 0.08 \text{ bar}$, $S_5 = 5\text{ bar}$. \(\Rightarrow h_5 = 173.8 \frac{\text{kJ}}{\text{kg}} \)

State 6: $h_6 = h_5 + h_5 (P_{eq} - P_5) = 18.88 + (1.0064 \times 10^3) \times (3 - 0.08) \frac{10^5}{10^5} = 174.7 \frac{\text{kJ}}{\text{kg}} \)

State 7: $P_7 = 3.6\text{ bar}$, $S_7 = 5\text{ bar}$. \(\Rightarrow h_7 = 5.61 \frac{\text{kJ}}{\text{kg}} \)

State 8: $h_8 = h_7 + h_7 (P_8 - P_7) = 5.61 \frac{\text{kJ}}{\text{kg}} \times 10^5 \times (60 - 3) \frac{10^5}{10^5} = 569.7 \frac{\text{kJ}}{\text{kg}} \)

State 9: $P_9 = 36\text{ bar}$, $T_9 = 205\text{K}$. \(\Rightarrow h_9 = 878.8 \frac{\text{kJ}}{\text{kg}} \)

State 10: $P_{10} = 20 \text{ bar}$, $S_{10} = 5\text{ bar}$. \(\Rightarrow h_{10} = 908.79 \frac{\text{kJ}}{\text{kg}} \)

State 11: $h_{11} = h_{10} = 908.79 \frac{\text{kJ}}{\text{kg}}$
Use energy rate balance for closed feedwater heater.

\[ 0 = y' (h_2 - h_{10}) + (h_y - h_g) \]

\[ \Rightarrow y' = \frac{h_g - h_y}{h_2 - h_{10}} = \frac{561.73 - 823.47}{-2983.5 - 908.79} = 0.1502 \]

For open feedwater heater.

\[ 0 = y'' h_3 + (1 - y'' - y''') h_b + y'' h_{11} - h_7 \]

\[ 0 = y'' \times 2589.6 + (1 - 0.1502 - y''') \times 1714.17 + 0.1502 \times 908.79 - 561.47 \]

\[ \Rightarrow y''' = 0.1147 \]

For turbine stages:

\[ \frac{w_t}{m} = (h_4 - h_3) + (1 - y'' y'') (h_b - h_{11}) = 1075.1 \frac{kJ}{kg} \]

For Pumps:

\[ \frac{w_p}{m} = (h_5 - h_4) + (1 - y''' y''') (h_b - h_{15}) = 8.47 \frac{kJ}{kg} \]

For Steam generator:

\[ \frac{G_{in}}{m} = h_1 - h_{10} = 2470.1 \frac{kJ}{kg} \]

The thermal efficiency is

\[ \eta = \frac{W_{net}}{Q_{in}} = \frac{1075.1 - 8.47}{2470.1} = 0.4315 = 43.18\% \]

Mass flow rate:

\[ \dot{m} = \frac{W_{net}}{h_1 - h_{10}} = \frac{(1075.1 - 8.47) 8.47}{2470.1} = 100 \text{ kW} \]

\[ \dot{m} = 93.75 \text{ kg/s} = 3.375 \times 10^5 \text{ kg/hr} \]
PROBLEM 3

\[ f(h) = \text{constant} I \]

\[ \dot{Q}_\text{in}/\dot{m} = \dot{Q}_\text{out}/\dot{m} \]

\[ \dot{Q}_\text{in} = (h_2 - h_3)(1 - \gamma) \]

\[ \dot{Q}_\text{in}/\dot{m} = h_1 - h_c \]

\[ \gamma = 1 - \frac{h_2 - h_3}{h_1 - h_c} \]

\[ R = \frac{h_c - h_3}{h_1 - h_c} = \frac{h_c - h_3}{\lambda} \]

where \( \lambda = h_1 - h_c \)

Define \[ D = \frac{h_1 - h_c}{1 - \gamma} \]

Energy Balance at OFH

\[ (1 - \gamma)h_4 + \gamma h_{cx} = h_c \Rightarrow h_4 - \gamma h_4 + \gamma h_{cx} = h_c \]

\[ \Rightarrow \gamma = \frac{h_c - h_4}{h_{cx} - h_4} \]

\[ \gamma = \frac{h_c - h_4}{h_{cx} - h_4} = \frac{h_{cx} - h_4}{h_{cx} - h_4} = \frac{h_c - h_4}{h_{cx} - h_4} \]
Make a reasonable first guess; $P_{int}$ must be between 0.05 bar and 0.08 bar. Know $S_f = S_{ex} = 6.6586$

Guess $P_{int} = 0.06$ bar

$S_f = 6.6586 \text{ KJ/m}^3$ $\Rightarrow$ $h_c = h_f (2811.5 - h_3) \Rightarrow h_{ex} = 2811.5 \text{ KJ/kg}$

$D = \frac{(h_f - h_{ex})(2811.5 - h_3)}{2811.5 - h_f} \Rightarrow D = 3329 \text{ (at 10 bar)} \quad R = 0.5$

Guess $P_{int} = 0.05$ bar

$S_f = 6.6586 \text{ KJ/m}^3$ $\Rightarrow$ $h_c = h_f (2811.5 - h_3) \Rightarrow h_{ex} = 2811.5 \text{ KJ/kg}$

$D = \frac{(h_f - h_{ex})(2811.5 - h_3)}{2811.5 - h_f} \Rightarrow D = 3322 \text{ (at 11 bar)} \quad R = 0.587$

Guess $P_{int} = 0.08$ bar

$S_f = 6.6586 \text{ KJ/m}^3$ $\Rightarrow$ $h_c = h_f (2811.5 - h_3) \Rightarrow h_{ex} = 2811.5 \text{ KJ/kg}$

$D = \frac{(h_f - h_{ex})(2811.5 - h_3)}{2811.5 - h_f} \Rightarrow D = 3328 \text{ (at 8 bar)} \quad R = 0.479$

One can assume that $D$ is well behaved (exponential) and has only one maximum. From the three values of $D$ found so far, it is apparent that the maximum occurs between 0.05 bar and 0.08 bar. Since the values of $D$ have gone up and then been down for increasing pressure.
So try a point in between. $P_{ex} = 10$ bar and $P_{ex} = 15$ bar.

Try 12.5 bar, though it maybe a little higher, works.

\[
\begin{align*}
P_{ex} &= 12.5 \text{ bar} \\
S_{ex} &= 6.5381 \text{ kJ/kJ} \\
\Rightarrow \quad h_{ex} &= 2853.9 \text{ kJ/kg}
\end{align*}
\]

\[
D = \frac{\left( h_i - 292.82 \right) \left( 2853.9 - h_j \right)}{2833.9 - 292.82} \Rightarrow D = 33.28 \text{ (at 12.5 bar)} 
\]

\[
\Rightarrow \quad \eta = 0.551
\]

So now we know that $D_{max}$ occurs between 10 bar and 17.5 bar. Let's try one more pressure, $P_{ex} = 11.25$ bar, since it's easy to interpolate between the steam chart using the values of $P_{ex} = 12.5$ bar.

\[
\begin{align*}
P_{ex} &= 11.25 \text{ bar} \\
S_{ex} &= 6.5381 \text{ kJ/kJ} \\
\Rightarrow \quad h_{ex} &= 2132.0 \text{ kJ/kg}
\end{align*}
\]

\[
D = \frac{\left( h_i - 292.82 \right) \left( 2132.0 - h_j \right)}{2833.9 - 292.82} \Rightarrow D = 33.28 \text{ (at 11.25 bar)} 
\]

\[
\Rightarrow \quad \eta = 0.533
\]

We are now well within the accuracy necessary to solve this problem. Let the pressure we have accomplished.
Thus, it seems \( R_{\text{opt}} \approx 0.51 \) and occur when \( D_{\text{max}} \approx 3729 \)

\[ R_{\text{opt}} \approx 0.51 \quad \text{51% regeneration} \]

Also, from the plot, \( R_{\text{opt}} \) occur when \( p \approx 10 \) bars

\[ \Rightarrow \quad (p_{\text{opt}} \approx 10 \text{ bars}) \]

\[ \Rightarrow \quad h_{\text{ex}} \approx 2012 \text{ kJ/kg} \]

\[ \Rightarrow \quad h_{\text{ex}} \approx 762 \text{ kJ/kg} \]

\[ \text{Condition for optimum extraction} \]

When \( f(h) = h_{\text{ex}} - h_{\text{he}} \) is assumed constant, the optimum extraction occur when \( R = 0.5 \). Notice how close this result is to the one found when \( h_{\text{ex}} - h_{\text{he}} \) is allowed to vary.

\[ f(h) \text{ is constant is a good approximation} \]

Note: there are several ways to solve this problem:

1) guessing - you can guess almost anything to make your way through the chart to get \( D \), I usually pen because it was using the charts as easy as possible. If one had more data or the same charts so not much interpolation would needed, this it would be most logical to guess \( R \) or \( h_{\text{he}} \) or \( h_{\text{ex}} \).

2) assume \( f(h) \) is either linear, quadratic, or cubic - this can be done by using the value of \((h_{\text{ex}}, h_{\text{he}}) + (h_{\text{ex}}, h_{\text{he}}))\) to obtain a linear approximation or \( f(h) \), or by using \((h_{\text{ex}}, h_{\text{he}}), (h_{\text{ex}}, h_{\text{he}}) + \text{the other two sets of} \) interpolation at the 2 extraction points in Problem #1 to obtain either a quadratic or cubic fit. However, this problem demonstrates that a quadratic or cubic curve fit is only very well, since assuming \( f(h) \) is constant only had an error of less than 2%.

\[ (f(h) = \text{constant}) \]
PROBLEM 4

State 1. \( P_1 = 2700 \text{ lb/ft}^2 \), \( h_1 = 1457.2 \text{ ft} \). \( S_1 = 1.5262 \text{ ft}^2 \). \( A_1 = 6.91 \).

State 2. \( P_2 = 600 \text{ lb/ft}^2 \), \( S_2 = 1.5262 \text{ ft}^2 \). \( A_2 = 1.2835 \text{ ft}^2 \).

State 3. \( P_3 = 60 \text{ lb/ft}^2 \), \( S_3 = 1.5262 \text{ ft}^2 \). \( A_3 = 0.9327 \text{ ft}^2 \). \( S_3 = 1.6743 \text{ ft}^2 \).

\[ T = \frac{S_3 - S_1}{S_3 - S_2} = \frac{1.6743 - 0.9327}{1.6743 - 0.7503} = 0.905 \]

\[ h_f = 0.62.2 \text{ ft}^2/\text{ft} \]

\[ h_f = 9.15.8 \text{ ft}^2/\text{ft} \]

\[ h_f = 2.62.2 \text{ ft}^2/\text{ft} \]

\[ h_f = 9.15.8 \text{ ft}^2/\text{ft} \]

\[ h_3 = h_f + \frac{S_3}{S_2} = 2.62.2 + 0.7503 \times 75 = 10 = 9.13 \text{ ft}^2/\text{ft} \]

State 4. \( P_4 = 141 \text{ lb/ft}^2 \), \( S_4 = 1.5262 \text{ ft}^2 \). \( A_4 = 0.9327 \text{ ft}^2 \). \( S_4 = 1.6743 \text{ ft}^2 \).

\[ h_f = 0.64.7 \text{ ft}^2/\text{ft} \]

\[ h_f = 0.7503 \text{ ft}^2/\text{ft} \]

\[ h_4 = h_f + \frac{S_4}{S_3} = 0.64.7 + 0.9327 \times 0.9327 = 8.52.13 \text{ ft}^2/\text{ft} \]

State 5. \( P_5 = 1 \text{ lb/ft}^2 \). \( S_5 = 1.5262 \text{ ft}^2 \).

\[ h_5 = 6.9 \text{ ft}^2/\text{ft} \]

State 6. \( P_6 = 60 \text{ lb/ft}^2 \), \( S_6 = 0.0164 \text{ ft}^2 \).

\[ h_6 = 9.15 \times (P_6 - P_5) = 69.74 \times 0.0164 \times \frac{1}{6} \]

\[ h_6 = 1.25 \text{ ft}^2/\text{ft} \]

State 7. \( P_7 = 60 \text{ lb/ft}^2 \), \( S_7 = 0.0164 \text{ ft}^2 \).

\[ h_7 = 2.622 \text{ ft}^2/\text{ft} \]

State 8. \( P_8 = 2500 \text{ lb/ft}^2 \), \( S_8 = 0.0173 \text{ ft}^2 \).

\[ h_8 = h_7 + \frac{S_8}{S_7} = 2.622 + 0.0173 \times \frac{5000 - 60}{60} \times 12 \text{ ft}^2/\text{ft} \]

\[ h_8 = 270.09 \text{ ft}^2/\text{ft} \]
State 9. \( P_9 = 2500 \text{ Btu/lb}, \quad T_9 = 428^\circ F \Rightarrow h_9 = 463.22 \text{ Btu/lb} \)

State 10. \( P_{10} = 600 \text{ Btu/lb}, \text{ sat.} \quad h_{10} = 471.7 \text{ Btu/lb} \)

State 11. \( h_{11} = h_{10} = 471.7 \text{ Btu/lb} \)

For the closed feedwater heater:

\[
y' = \frac{h_9 - h_8}{h_3 - h_{10}} = \frac{463.22 - 279.09}{1283.5 - 471.7} = 0.2378
\]

For the open feedwater heater:

\[
y'' = \frac{(h_9 - h_8) + (1-y')y'(h_8 - h_{11})}{h_3 - h_6} = \frac{h_7 - h_6 + y'(h_6 - h_{11})}{h_3 - h_6}
\]

\[
y'' = \frac{262.6 - 69.92 + 0.2379(69.92 - 471.7)}{1089.12 - 69.92}
\]

\[
y'' = 0.0958
\]

\[
\frac{U^+}{m} = (h_1 - h_2) + (1-y')(h_3 - h_6) + (1-y''y')(h_3 - h_4)
\]

\[
= (1457.2 - 1283.5) + (1 - 0.2379)(1283.5 - 1089.12) + (1 - 0.2379 - 0.0958)(1089.12 - 862.18)
\]

\[
= 479.86 \text{ Btu/lb}
\]

\[
\frac{W_r}{m} = (h_8 - h_7) + (1-y''y')(h_6 - h_9)
\]

\[
= (270.09 - 262.2) + (1 - 0.2379 - 0.0958)(69.92 - 69.92)
\]

\[
= 8.01 \text{ Btu/lb}
\]

\[
\frac{Q_{in}}{m} = h_1 - h_q = 1457.2 - 463.22 = 993.98 \text{ Btu/lb}
\]

\[
\eta = \frac{U^+ - W_r}{Q_{in}} = \frac{479.86 - 8.01}{993.98} = 47.5 \%
\]