ME 251- RECITATION # 8

Instructor: R. Betti
TAs: A. Chirstopherson, J. Cramer, R. Mariuz, X. Zhou

November 7, 2013; Gavet 310; 4:50-6:05 PM

SAMPLE PROBLEMS FOR MIDTERM EXAM # 2

[1] Air enters the compressor of a gas turbine at 100 KPa, 300 K. The air is compressed in two stages to 900 kPa, with intercooling to 300 K between stages at a pressure of 300 kPa. The turbine inlet temperature is 1480 K and the expansion occurs in two stages, with reheat to 1420 K between the stages at a pressure of 300 kPa. The compressor and turbine stage efficiencies are 84 and 82% respectively. The net power developed is 1.8 MW. Determine

(a) The volumetric flow rate in, m$^3$/s at the inlet of each compressor stage.
(b) The thermal efficiency of the cycle
(c) The back work ratio (compressor work/turbine work)

Repeat this problem after adding a regenerator with an effectiveness of 75%.

[2] Derive the relations for isentropic flows between
(a) pressure and temperature
(b) pressure and specific volume
(c) pressure and density
(d) temperature and density
(e) temperature and specific volume
Use the cold air analysis with constant specific heats

[3] Derive the optimum intercooling pressure that minimizes the work of a compressor used to compress air from an initial pressure P1 to a final pressure P2. Use the cold air analysis with constant specific heats.
1) Solution

**Schematic & Given Data:**

<table>
<thead>
<tr>
<th>State</th>
<th>Conditions</th>
<th>States</th>
<th>Turbine State</th>
<th>Compressor State</th>
<th>Condenser State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$T_1 = 300$ K, $h_1 = 300.19$ kJ/kg, $P_{r1} = 1.8860$</td>
<td>$h_a = h_{c1} = 84$</td>
<td>$h_{c1} = h_{a} + \frac{(h_{a} - h_{1})}{\eta_{c}} = 432.42$ kJ/kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$P_{r2} = \frac{(300/1000) \times 1.8860}{4.158} = 4.158$</td>
<td>$h_{c2} = h_{a} + \frac{(h_{a} - h_{1})}{\eta_{c}} = 432.42$ kJ/kg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$T_3 = 1480$ K, $h_3 = h_{c1} - 84$</td>
<td>$h_{c1} = h_{a} + \frac{(h_{a} - h_{1})}{\eta_{c}} = 432.42$ kJ/kg</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$T_4 = 1420$ K, $h_4 = h_{a} + \frac{(h_{a} - h_{1})}{\eta_{c}} = 432.42$ kJ/kg</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Analysis:**

First, fix each of the principal states. (Table A-22)

**State 1**

$T_1 = 300$ K $\Rightarrow$ $h_1 = 300.19$ kJ/kg, $P_{r1} = 1.8860$

**State 2**

For isentropic compression, $P_{r2} = \frac{(300/1000) \times 1.8860}{4.158} = 4.158$. Thus $h_{c1} = 432.42$ kJ/kg.

**State 3**

$h_{c1} = h_{a} - h_{1} = 300.19$ kJ/kg, $P_{rb} = 1.8860$

**State 4**

$h_{c2} = h_{a} + \frac{(h_{a} - h_{1})}{\eta_{c}} = 432.42$ kJ/kg

**State 5**

For isentropic expansion, $P_{r3} = \frac{(P_{c}/P_{g})}{P_{r3}} = 189.60 \Rightarrow h_{c5} = 1201.5$ kJ/kg

Using the Turbine Stage efficiency,

$h_{c1} = h_{a} - h_{1} = 300.19$ kJ/kg, $P_{rb} = 1.8860$

**State 6**

$h_{c6} = h_{a} - h_{1} = 300.19$ kJ/kg, $P_{rb} = 1.8860$

(a) To determine the volumetric flow rate, first find $m$ using

\[
\frac{\dot{W}_{cycle}}{m} = (h_{3} - h_{c}) + (h_{c} - h_{4}) - (h_{a} - h_{1}) = 394.6$ kJ/kg

\[
\dot{m} = \frac{(1800 \text{ kW})}{(394.6 \text{ kJ/kg})} = \frac{(1800 \text{ kJ/s})}{(394.6 \text{ kJ/kg})} = 4.562 \text{ kg/s}
\]

Then, $\frac{\dot{V}_{1}}{m} = \frac{4.562 \text{ kg/s}}{300 \text{ K}} = 1.52 \text{ m}^3/\text{s}$
\[ (AV)_b = \frac{\text{in}^2 \text{ft}^2}{\text{ft}^3} \times (4.36) \times \left( \frac{3274}{20.49} \right) \times \left( \frac{300}{3 \text{ ft/s}} \right) = 1.31 \text{ m}^3 \text{s}^{-1} \]

(a) The thermal efficiency is obtained as follows:

The total heat added is
\[ \dot{Q}_{\text{in}} = (h_3 - h_2) + (h_d - h_6) = 1443.41 \text{ kJ/kg} \]

Thus
\[ n = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_{\text{in}}} = \frac{394.46}{1443.41} = 0.2734 (27.34\%) \]

(c) The back work ratio is
\[ \text{bwr} = \frac{\dot{W}_{\text{cycle}}}{\dot{W}_{\text{in}}} = \frac{(h_6 - h_7) + (h_2 - h_5)}{(h_3 - h_5) + (h_d - h_4)} = \frac{264.40}{659.06} = 0.401 \]

}\[ \text{bwr} \]

### Schematic & Given Data

![Schematic Diagram]

**Assumptions:** Same as in Problem 9.73. Also, there is no pressure drop across the regenerator.

**Analysis:** From the solution to Problem 9.73.

\[ h_1 = h_6 = 306.19 \text{ kJ/kg} \]
\[ h_2 = 432.42 \text{ kJ/kg} \]
\[ h_3 = 432.42 \text{ kJ/kg} \]
\[ h_4 = 1210.77 \text{ kJ/kg} \]
\[ h_5 = 1111.79 \text{ kJ/kg} \]

State x Using the regenerator effectiveness
\[ \eta_{\text{reg}} = \frac{h_x - h_6}{h_x - h_5} \Rightarrow h_x = \eta_{\text{reg}} (h_4 - h_5) + h_5 = 1082.08 \text{ kJ/kg} \]

(a) The net work of the gas turbine is unchanged. Thus
\[ \dot{W}_{\text{cycle}} = 394.46 \text{ kJ/kg} \]

Since the states at 1 and 6 remain the same, the volumetric flow rates also remain unchanged: \((AV) = 388 \text{ ft}^3/\text{s}, (AV)_b = 1.31 \text{ m}^3/\text{s} \)

(b) However, the heat addition becomes
\[ \dot{Q}_{\text{in}} = (h_3 - h_2) + (h_d - h_6) = 855.15 \text{ kJ/kg} \]

Thus
\[ n = \frac{394.46}{855.15} = 0.461 (46.1\%) \]

(c) The back work ratio also is unchanged.
\[ \text{bwr} = 0.401 \]

1. As a result of the introduction of the regenerator, there is a significant increase in thermal efficiency.
SOLUTION 2)

Isentropic flow equations for cold air analysis with constant specific heats

First law: \( dh = T ds + v dp \)
Isentropic flow: \( ds = 0 \), \( s = \text{constant} \)
Constant specific heat: \( dh = c_p \, dT \)
Ideal gas equation of state: \( pv = RT \) \( (R = R_0/M) \)

Rewrite first law: \( c_p \, dT = RT \, (dp/p) \) or \( dT/T = R/c_p \, (dp/p) \)

Use \( R = c_p - c_v \) and \( k = c_p / c_v \).

Integrate first law:

\( (T/T_0) = (P/P_0)^{(k-1)/k} \)

replace \( T \) with \( pv/R \) to find

\( (P/P_0) = (v_0/v)^k \)

replace \( v = 1/\rho \) to find

\( (P/P_0) = (\rho/\rho_0)^k \)

SOLUTION 3)

Compressor work for two stage compression from \( P_1 \) to \( P_2 \) with intercooling back to initial temperature \( T_1 \). Find intermediate pressure \( P_i \) of the intercooling phase that minimized the total compressor work:

\( W(1 \rightarrow 2) = h_2 - h_1 + h_i - h_1 = c_p (T_2 - T_1 + T_i - T_1) = c_p T_1 [(T_2/T_1) + (T_i/T_1) - 2] \)

Use \( (T_2/T_1) = (P_2/P_i)^{(k-1)/k} \) and \( (T_i/T_1) = (P_i/P_1)^{(k-1)/k} \)

Rewrite:

\( W(1 \rightarrow 2) = h_2 - h_1 + h_i - h_1 = c_p (T_2 - T_1 + T_i - T_1) = c_p T_1 [(P_2/P_i)^{(k-1)/k} + (P_i/P_1)^{(k-1)/k} - 2] \)

Minimize \( W \) by setting derivative to zero: \( dW/dP_i = 0 \)

Find optimum intercooling pressure: \( P_i = \sqrt{P_1 \, P_2} \)