[1] (from HW 1) Air enters a compressor operating at steady state with a pressure of 14.7 lbf/in.$^2$, a temperature of 80°F, and a volumetric flow rate of 1000 ft$^3$/min. At the exit, the pressure is 100 lbf/in.$^2$ and the temperature is 400°F. Using the ideal gas model, determine the diameter of the exit duct so the exit air velocity does not exceed 700 ft/s.

[2] (from HW 1) Steam enters a turbine operating at steady state at 600°F and 200 lbf/in.$^2$ with a velocity of 80 ft/s and leaves as saturated vapor at 5 lbf/in.$^2$ with a velocity of 300 ft/s. The power developed by the turbine is 200 horsepower. Heat transfer from the turbine to the surroundings occurs at a rate of 50,000 Btu/h. Neglecting potential energy effects, determine the mass flow rate of steam, in lb/s.

[3] (not from HW) Air expands through a turbine from 10 bar, 900 K to 1 bar, 500 K. The inlet velocity is small compared to the exit velocity of 100 m/s. The turbine operates at steady state and develops a power output of 3200 kW. Heat transfer between the turbine and its surroundings and potential energy effects are negligible. Calculate the mass flow rate of air, in kg/s, and the exit area, in m$^2$.

[4] (not from HW): Steam enters a nozzle operating at steady state at 30 bar, 320°C, with a velocity of 100 m/s. The exit pressure and temperature are 10 bar and 200°C, respectively. The mass flow rate is 2 kg/s. Neglecting heat transfer and potential energy, determine

(a) the exit velocity, in m/s.
(b) the inlet and exit flow areas, in cm$^2$. 
1). Solution

\[ P_1 = 147 \text{ lb/ft}^2 \]
\[ T_1 = 80^\circ F = 539.67^\circ R \]
\[ V_1 = 1000 \text{ ft}^3/\text{min} \]

\[ P_2 = 100 \text{ lb/ft}^2 \]
\[ T_2 = 400^\circ F = 859.67^\circ R \]
\[ V_2 = 700 \text{ ft/s} \]

Because we use the ideal gas model, at steady state,
\[ \frac{dm}{dt} = 0 = m_1 - m_2 = 0 \Rightarrow m_1 = m_2 \]

Thus \[ \frac{A_1 \cdot V_1}{V_1} = \frac{A_2 \cdot V_2}{V_2} \Rightarrow A_2 = \frac{A_1 \cdot V_1 \cdot \frac{V_2}{V_2}}{V_1 \cdot \frac{V_2}{V_2}} = \frac{V_1}{V_2} \cdot \frac{V_2}{V_1} \]

Ideal gas \[ \Rightarrow P_1 V_1 = m \cdot RT_1 \Rightarrow \frac{V_1}{m} = \frac{RT_1}{P_1} = \gamma_1 \]
\[ P_2 V_2 = m_2 \cdot RT_2 \Rightarrow \frac{V_2}{m_2} = \frac{RT_2}{P_2} = \gamma_2 \]

Thus \[ \frac{V_2}{V_1} = \frac{V_2}{m_2} \cdot \frac{m_1}{V_1} = \frac{RT_2}{P_2} \cdot \frac{RT_1}{P_1} = \frac{T_2 \cdot P_1}{T_1 \cdot P_2} \]
\[ \frac{V_2}{V_1} = \frac{859.67^\circ R \times 14.7 \text{ lb/ft}^2}{539.67^\circ R \times 100 \text{ lb/ft}^2} = 0.234 \]

\[ A_2 = \frac{V_1}{V_2} \cdot \frac{V_2}{V_1} = \frac{1000 \text{ ft}^3/\text{min}}{700 \text{ ft/s}} \times 0.234 = \frac{1000 \times 0.234}{700 \times 60} = 5.58 \times 3 \]

Thus \[ D_2 = 2 \cdot \sqrt{\frac{A_2}{\pi}} = 0.084 \text{ ft} = 1.01 \text{ inches} \]

Diameter of the exit should be at least 1.01 inches.
2). Solution.

\[
\begin{align*}
T_1 &= 600\, ^\circ F \\
P_1 &= 200\, \text{lb f/sq in} \\
v_1 &= 80\, \text{ft/s} \\
T_2 &= \text{Saturated Vapor} \\
P_2 &= 5\, \text{lb f/sq in} \\
v_2 &= 300\, \text{ft/s} \\
\dot{W}_{cv} &= 200\, \text{hp} = 200\, \text{hp} \cdot \frac{2545\, \text{Btu/h}}{\text{hp}} = 50\% \\
\dot{Q} &= 50,000\, \text{Btu/h} \\
\end{align*}
\]

Assume:
1. Steady State;

Use Energy Rate Balance:

\[
0 = \dot{Q} - \dot{W}_{cv} + m\left(\bar{h}_1 - \bar{h}_2\right) + \frac{1}{2}m\left(v_1^2 - v_2^2\right) + m\left(g\left(z_1 - z_2\right)\right)
\]

\[
= -50,000\, \text{Btu/h} - 509,000\, \text{Btu/h} + m\left[\bar{h}_1 - \bar{h}_2 + \frac{v_1^2 - v_2^2}{2}\right]
\]

Use Table A-3E for \(h_2 = 1131.0\, \text{Btu/lb}\),

A-4E for \(h_1 = 1322.1\, \text{Btu/lb}\).

Solve for \(m\):

\[
m = \frac{559,000\, \text{Btu/h}}{(1322.1-1131.0)\, \text{Btu/lb} + \frac{(80^2 - 300^2)}{2} - \frac{32176 \times 778.17}{2 \times 32.176 \times 778.17} \times \frac{1\, \text{Btu}}{\text{lb}}}
\]

\[
m = 2.951.0\, \text{lb/h} = 0.82\, \text{lb/s}
\]

The mass flow rate of steam is 0.82 lb/s.
3) Solution

**SCHEMATIC & GIVEN DATA:**

\[ P_1 = 10 \text{ bar} \]
\[ T_1 = 900 \text{ K} \]
\[ V_1 < V_2 \]
\[ W_{cv} = 3200 \text{ kW} \]

\[ P_2 = 1 \text{ bar} \]
\[ T_2 = 500 \text{ K} \]
\[ V_2 = 100 \text{ m/s} \]

**ASSUMPTIONS:** (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Potential energy effects and kinetic energy at the inlet can be neglected. (4) The air behaves as an ideal gas.

**ANALYSIS:** Begin with a steady-state energy balance

\[ 0 = Q_{cv} - W_{cv} + \dot{m} \left[ (h_1 - h_2) + \frac{V_2^2 - V_1^2}{2} + g \left( z_1 - z_2 \right) \right] \]

where \( \dot{m}_1 = \dot{m}_2 = \dot{m} \). Solving for \( \dot{m} \)

\[ \dot{m} = \frac{W_{cv}}{(h_1 - h_2) - \frac{V_2^2}{2}} \]

From Table A-22; \( h_1 = 932.93 \text{ kJ/kg} \) and \( h_2 = 503.02 \text{ kJ/kg} \). Thus

\[ \dot{m} = \frac{(3200 \text{ kW}) \left( 1 \text{ kW} / \text{ kJ} \right)}{(932.93 - 503.02) \text{ kJ/kg} - \left( \frac{100^2 \text{ m}^2}{2} \right) \left( \frac{1 \text{ N} \cdot \text{m/s}^2}{10^5 \text{ N/m}^2} \right) \left( 1 \text{ kJ} / 10^3 \text{ N} \cdot \text{m} \right) \] \]

\[ \dot{m} = 7.53 \text{ kg/s} \]

The exit area is

\[ A_2 = \frac{V_2 \dot{m}}{\sqrt{T_2}} = \frac{RT_2 \dot{m}}{P_2 V_2} \]

\[ = \frac{(8.314 \text{ kJ/kg} \cdot \text{K}) \cdot (500 \text{ K}) \cdot (7.53 \text{ kg/s})}{(1 \text{ bar}) \cdot (100 \text{ m/s})} \]

\[ = 0.108 \text{ m}^2 \]
4) Solution

**SCHEMATIC & GIVEN DATA:**

\[ P_1 = 30 \text{ bar} \quad T_1 = 320^\circ \text{C} \]

\[ V_1 = 100 \text{ m/s} \quad m = 2 \text{ kg/s} \]

\[ T_2 = 200^\circ \text{C} \quad P_2 = 10 \text{ bar} \]

**ASSUMPTIONS:**

1. The control volume is at steady state.
2. Heat transfer is negligible and \( W_{in} = 0 \).
3. Potential energy effects are negligible.

**ANALYSIS:**

(a) The velocity of steam at the exit is found from the steady-state energy balance:

\[ \dot{Q}_{in} - \dot{Q}_{out} + m \left[ (h_1 - h_2) + \frac{(T_1 - T_2)}{1} + g \cdot \frac{z_1 - z_2}{x} \right] \]

where \( m_1 = m_2 = m \). Solving for \( V_2 \):

\[ V_2 = \sqrt{2 \left( h_1 - h_2 \right)} + \frac{V_1^2}{2} \]

From Table A-4, \( h_1 = 3043.4 \text{ kJ/kg} \) and \( h_2 = 2827.9 \text{ kJ/kg} \). Thus

\[ V_2 = \sqrt{2 \left( 3043.4 - 2827.9 \right) \frac{\text{kJ}}{\text{kg}}} \left[ \frac{1 \text{ kg-m/s}^2}{1 \text{ N}} \right] \left[ \frac{10^3 \text{ N-m}}{1 \text{ kJ}} \right] + (100^2) \text{m}^2/\text{kg}^2 \]

\[ V_2 = 664.1 \text{ m/s} \]

(b) To find the inlet and exit flow areas, use \( m = \dot{m} = V \). Solving:

\[ A_1 = \frac{m V_1}{V_1} \quad \text{and} \quad A_2 = \frac{m V_2}{V_2} \]

From Table A-4, \( T_1 = 0.0850 \text{ m}^3/\text{kg} \) and \( V_2 = 0.2060 \text{ m}^3/\text{kg} \). Thus

\[ A_1 = \frac{(2 \text{ kg/s})(0.0850 \text{ m}^3/\text{kg})}{10^4 \text{ cm}^2/1 \text{ m}^2} = 17 \text{ cm}^2 \]

\[ A_2 = \frac{(2)(0.2060)}{(664.1)} \left[ \frac{10^9}{1} \right] = 6.2 \text{ cm}^2 \]