# ME 201/MTH 281/ME 400/CHE 400 EXAM #1

## THURSDAY OCTOBER 8, 2009 2:00 – 3:15 PM and 3:25 – 4:40 PM

This exam covers assignments 1 through 4, and Chapters 1 and 2 and section 3.1 of the class notes. You may use any books, notes or other references, but you may not exchange material with anyone else. If you need additional information to work a problem, ask me for it, and, if it is appropriate, I will put it on the board. Do all four problems. **Be sure to explain your work! Wrong calculations with no explanation will receive very little partial credit.** Solutions will be posted on the web when both exams are over, and printed solutions will be distributed in class on Friday Oct. 9. Graded exams will be returned in class on Monday October 12. Exam statistics will be posted on the web. Good luck!

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**(1)** (25 points) Consider the modified one-dimensional heat conduction problem given below in which $\gamma$, $\beta$ and $T_0$ are all positive constants and $D(x)$ is a continuous and positive function. Look for separated solutions of the form $F(x)G(t)$. Find the equations satisfied by $F$ and $G$, and the boundary conditions satisfied by $F$. Do NOT try to solve the equations for $F$ and $G$.

$$\frac{\partial T}{\partial t} = D(x)\frac{\partial^2 T}{\partial x^2} - \gamma e^{-\beta t} T, \quad 0 < x < L,$$

$$\text{with } T(0,t) = 0, \quad \frac{\partial T}{\partial x}(L,t) = 0, \text{ and } T(x,0) = T_0(x/L).$$

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**(2)** (25 points) Consider the function $f(x) = (1-x)^2$. For each series below, sketch three periods of the periodic function represented by the series, and tell how rapidly the Fourier coefficients $a_n$ and/or $b_n$ will decrease with $n$. You do NOT need to calculate any Fourier coefficients to answer these questions.

**(a)** Full Fourier series of $f(x)$ on $-1 \le x \le 1$.

**(b)** Fourier cosine series of $f(x)$ on $0 \le x \le 1$.

**(c)** Fourier sine series of $f(x)$ on $0 \le x \le 1$.

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**((3)** (35 points) Solve the initial value problem given below, in which $D$, $\gamma$, and $T_0$ are all positive constants. You may use any results derived in class, but be sure to explain your work.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + \gamma, \quad 0 < x < L, \ t > 0,$$

$$\text{with } T(0,t) = 0, \quad T(L,t) = 0, \text{ and } T(x,0) = \frac{\gamma}{2D}x(L-x) + T_0 \sin\left(\frac{\pi x}{L}\right).$$

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**(4)** (15 points) A squash player is attempting to warm up a squash ball before play by immersing it in a basin of well-stirred hot water. The initial temperature of the ball is 20 ˚C and the water in the basin is at 50 ˚C. For purposes of this problem, take the squash ball to be solid rubber, with diameter 5 cm, and with a thermal diffusivity of $5 \times 10^{-8}$ m$^2$/s. The player soaks the ball for two minutes. Use what you know about diffusion times to comment on the efficacy of this process.

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