ME 201/MTH 281/ME 400/CHE 400
ASSIGNMENT #4 2009

Assignments handed in by 6 PM on Wednesday Sept. 30 will receive a 5 point bonus. Assignments handed in after that but by 6 PM on Thursday will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday Oct. 1.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
2. Fourier Series		
2.4 Fourier Sine and Cosine Series	W Sept. 23	3.3
2.5 Separation of Variables Revisited	Th Sept. 24	Chapter 2
3. Separation of Variables, Part I		
3.1 Diffusion Equation	F, M Sept. 25, 28	8.2

PROBLEMS
2.4 FOURIER SINE AND COSINE SERIES

(1) (21 points) Consider the function $f(x) = 2x - x^2$ **.** In each of the three parts below, find explicitly the series asked for, sketch the periodically extended function represented by the series on [-3,3], and show that the rate of convergence of the series is consistent with the smoothness of the periodically extended function. Tell what the series converges to at each point of the given interval.

(a) (7 points) The full Fourier series for $f(x)$ on $-1 \le x \le 1$.
(b) (7 points) The Fourier sine series for $f(x)$ on $0 \le x \le 1$.
(c) (7 points) The Fourier cosine series for $f(x)$ on $0 \le x \le 1$.

2.5 SEPARATION OF VARIABLES REVISTED AND 3.1 DIFFUSION EQUATION

(2) (14 points) If there were no fluid motions in the earth's core, the magnetic field **B** in the core would be governed by the diffusion equation: $\frac{\partial \mathbf{B}}{\partial t} = \lambda \nabla^2 \mathbf{B}$, where $\lambda = 2.7$ m^2/s is the magnetic diffusivity in the core. The radius of the core is $R = 3.5 \times 10^6$ m. Use the concept of the diffusion time developed in class to estimate how long it would take the magnetic field to diffuse out of the core under these conditions. Is your answer consistent with the paleomagnetic evidence which shows that the earth has had a magnetic field for a time of the order of 10^9 years? What is your conclusion?

(3) (65 points) Consider the initial value problem given below for $T(x,t)$.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + \beta \sin(\pi x / L) \quad , \quad 0 < x < L \; , t > 0 \; , T(0,t) = 0 \; , T(L,t) = 0 \; , T(x,0) = T_0 \; ,$$

where β and T_0 are constants. This is the equation for transient heat conduction in a bar of length *L*, with the ends maintained at zero temperature, and with a heat source of strength $\Gamma = \rho c \beta \sin(\pi x / L)$.

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(3) (continued) **(a)** (10 points) Follow the procedure given in class, and split the solution T into a steady-state part T_s and a transient part $\hat{T}$. Give a complete formulation (equation, boundary conditions, and, for the transient, an initial condition) for the determination of T_s and $\hat{T}$.

(b) (10 points) Find the steady-state solution T_s.

(c) (10 points) Find the transient solution $\hat{T}$. You may make use of any results obtained in class, but be sure to explain your work.

(d) (5 points) Verify that your solution T approaches the steady-state solution as time t goes to infinity. On the basis of this observation, plus any others you might want to make about the solution, would you agree or disagree with the following somewhat vague statement?

Solutions of the diffusion equation remember their initial conditions.

(e) (10 points) Sometimes we want to have more information than the simple result of (b) gives us – for example, an approximate solution which shows the approach to steady state. Show that for large times, the series solution for the transient $\hat{T}$ reduces to a simple one-term approximation. Estimate how large the time must be for this approximation to be valid.

(f) (10 points) For the following parameter values, estimate the time at which the temperature and the steady-state temperature differ by less than $\pm 2°\text{C}$ throughout the bar: $T_0 = 30°\text{C}$, L = 0.1 m, $\beta = 4.8$ °C/s, and $D = 2.0 \times 10^{-4}$ m^2/s.

(g) (10 points) For the parameter values given in (f), use Mathematica to construct some graphs of T versus x for values of time t at one-second intervals from $t = 0$ s to $t = 15$ s. Use the known initial values for $t = 0$, and use the first five nonzero terms in the transient series for the other graphs.

CHALLENGE PROBLEM

Solve the boundary value problem given below by separation of variables. The equation describes transient heat flow in a long bar in which heat is (1) conducted through the cross-section of the bar (the term $D\partial^2 T / \partial x^2$) and (2) lost to the surroundings by convection from the surface of the bar, at a rate γt which increases with time. Here D and γ are positive constants.

$$\frac{\partial T}{\partial T} = D\frac{\partial^2 T}{\partial x^2} - \gamma t T,\ 0 < x < L \text{ and } t > 0,$$

$$\text{with } T(0,t) = 0,\ T(L,t) = 0, \text{ and } T(x,0) = T_0 \frac{x}{L}\left(1 - \frac{x}{L}\right).$$