ME 201/MTH 281/ME 400/CHE 400
ASSIGNMENT #1 2009

Assignments handed in by 6 PM on Wednesday September 9 will receive a 5-point bonus. Assignments handed in after that but by 6 PM on Thursday September 10 will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday. Most of this assignment is review material from MTH 163 (or MTH 165) and MTH 164.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
1.1 Continuous Systems and Partial Differential Equations	W Sept. 2	1.1
1.2 Heat Conduction and the Diffusion Equation	Th Sept. 3	1.2-1.5

HOMEWORK PROBLEMS
REVIEW PROBLEMS FROM MTH 164

(1) (15 points) Let **H** be the vector field $\mathbf{H}(x,y,z) = (2x-y)\mathbf{i} + (z-x-4y)\mathbf{j} + (y+2z)\mathbf{k}$.
(a) (5 points) Calculate $\nabla \cdot \mathbf{H}$.
(b) (5 points) Show that $\nabla \times \mathbf{H} = 0$. We know from vector calculus that if the curl of a vector field is zero everywhere, then there exists a scalar function Φ such that $\mathbf{H} = \nabla\Phi$. Find Φ in this case.
(c) (5 points) Show that this Φ satisfies the equation $\nabla^2\Phi = 0$. This is called Laplace's equation and we will spend considerable time in this course learning how to solve it.

(2) (10 points) Consider the vector field $\mathbf{M} = \mathbf{H} + x\mathbf{i}$ where **H** is the same as in problem (1). Let S be the surface of the sphere with center at the origin and radius 2. Let **n** be the unit exterior normal to S, and let $d\sigma$ be the element of area on S. Show that $\oiint_S \mathbf{M} \cdot \mathbf{n}\, d\sigma = \frac{32\pi}{3}$.

(3) (10 points) Let C be the circle $x^2 + y^2 = 1$ in the plane $z = 1$. In the integral below, the circle is traversed counterclockwise when viewed from above the plane $z = 1$, and $d\mathbf{s}$ is the vector element of arc along C. Let **G** be the vector field $z\mathbf{H}$ where **H** is the vector field given in problem (1). Evaluate in any way you like the integral $\oint_C \mathbf{G} \cdot d\mathbf{s}$.

(4) (15 points) The temperature distribution in a region is given by $T(x,y) = T_0 + ax + by$, where $T_0 =$ 15 ˚C, $a = 30$ ˚C/km, and $b = -20$ ˚C/km. You may think of x and y as map coordinates, with x being positive to the east, and y being positive to the north.
(a) (5 points) Find the rate of change of temperature with distance in the direction northeast and show that it is the same at all points.
(b) (5 points)A hiker walks northeast at a constant speed. The hiker carries a recording thermometer and measures the change of temperature with time to be 0.5 ˚C /minute. How fast is he walking?

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(4) (continued) (**c**) (5 points) If another hiker walks at the same speed to the northwest, what rate of change of temperature with time does she measure?

REVIEW PROBLEMS FROM MTH 163 OR 165

(5) (13 points) Solve the initial-value problem $\frac{dx}{dt} + \frac{1}{t}x = 1$, $x(1) = 1$.

(6) (13 points) Solve the initial-value problem $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0$, $x(0) = 0$, $\frac{dx}{dt}(0) = 3$.

(7) (12 points) Solve the initial-value problem $\frac{d^2y}{dx^2} + 16y = 0$, $y(0) = 0$, $\frac{dy}{dx}(0) = 8$.

(8) (12 points) Solve the initial-value problem $\frac{d^2y}{dx^2} - 4y = 0$, $y(0) = 0$, $\frac{dy}{dx}(0) = 2$.

CHALLENGE PROBLEM

This problem is more of a preview of work in this course than a review of earlier work, although the computational aspects should be familiar from your previous work in differential equations. At this point in the course, the problem will seem somewhat unmotivated. The motivation and theoretical framework for the problem will become much clearer later. The topic of the problem is a particular two-point boundary value problem. This is a problem in which we specify conditions for a differential equation at two distinct points rather than specifying initial conditions at a single point.

(a) Consider the two-point boundary value problem for $y(x)$ given below. The quantity λ appearing in the equation is a positive constant. Are there any values of λ for which the problem does not have a solution?

$$\frac{d^2y}{dx^2} + \lambda y = 0 \text{ , } y(0) = 1 \text{ , } y'(1) = 0 \text{ .}$$

(b) Consider the homogeneous two-point boundary value problem given below, with the same equation but now with homogeneous boundary conditions. As in part (a), λ is a positive constant. Are there any values of λ for which there is a non-trivial solution – i.e., a solution which is not identically zero?

$$\frac{d^2y}{dx^2} + \lambda y = 0 \text{ , } y(0) = 0 \text{ , } y'(1) = 0 \text{ .}$$