

THE AMBASSADOR'S APPETIZER

**ME 201/MTH 281
PROJECT 2000**



GENERAL DESCRIPTION OF PROJECT

In this project, you are on a training exercise for Starfleet Cadets. The exercise begins aboard the USS Enterprise, which is in the Glia starsystem orbiting the planet Glia-2. The exercise, which is supposed to be routine, is suddenly altered by an event which is catastrophic for the Klingon Ambassador, and potentially catastrophic for the fragile peace between the Federation and the Klingons. Your unlikely task is to avert an Intergalactic war by analyzing theoretically an event in the Enterprise's kitchen. The details of all this are given below.

PRACTICAL MATTERS

You may work alone, or in groups of two. You may discuss the project freely with others. If you get significant help from someone else, that fact should appear as an acknowledgment or reference in your report. Any books or articles that you use also should be referenced. Your report should be brief but self-contained, so that a reader not having access to these directions will know what you have done.

A typical report organization might be something like the following: Introduction (with a general description of the problem); Formulation (the detailed quantitative formulation of the model used to make predictions); Results (results of the model calculations, including any graphs or tables necessary to make the results clear); Summary (a summary of your conclusions); References (books, articles or people consulted). Although typed reports are preferred, neatly handwritten reports are acceptable. Your project grade will be based on both the write-up (20%) and the technical content (80%). For groups of two, only one report need be submitted.

DETAILED DESCRIPTION OF PROJECT PROBLEM

Background on the Glia System

Visits of the USS Enterprise to the Glia starsystem began on Stardate 5526.3, with an examination of an ecosystem on Glia-2 (ME163 F93). On that visit Starfleet Academy Cadets saved an ecosystem. They were commended even though their bold plan violated the Prime Directive. The second visit to the Glia system, on Stardate 6872.1, involved the launch of an unmanned probe to the large airless planet Glia-4 (ME 163 F94). Showing great ingenuity, the Cadets on that mission successfully designed a nonlinear landing suspension for the probe. The discovery on Glia-4 of naturally occurring dilithium crystals (used in warp propulsion) led to the third visit on stardate 7304.6, during which Academy Cadets solved a crucial forced vibrations problem, allowing the dilithium crystals to be transported undamaged to the Enterprise (ME163 S97). Because of the importance of the dilithium mining, a colony in a biodome was set up on Glia-4 to continue the mining operation. An epidemic of Jasmine fever on Glia-4 was the occasion of the fourth visit. Once again Academy Cadets proved the value of mathematics in modeling real events, and developed a vaccine transport strategy which minimized the number of lives lost (ME 163 S98). The fifth adventure in the Glia system (stardate 8378.2, ME163 S99) began innocently as a recreational visit to the abandoned colonies on Glia-6. The situation was instantly transformed into a dire emergency when Captain Kirk was bitten by a feral glion carrying a deadly virus. Only by the most strenuous efforts were the Academy Cadets able to save Kirk, by developing an optimal treatment protocol for the viral infection. The sixth training mission to the Glia starsystem (Stardate 8993.1, ME 163 S2000) required Cadets to calculate re-entry orbits for a 20th century Apollo capsule, and then, in a surprise test of their computational acumen, to actually ride the capsule down to the surface of Glia-3, accompanied by Professor Clark's cat Billie. Additional details on some of these earlier visits to the Glia system are available on the web at

<http://www.me.rochester.edu:8080/courses/ME163/webproj/proj.html>. The present seventh visit is the first involving upper class cadets from Starfleet Academy.

Who is on the Enterprise?

In addition to the 283 crew members, the Enterprise is carrying two Cadets from Star Fleet Academy, all with advanced training in partial differential sciences, and you are one of those two Cadets. You note with some interest the unusual presence of the Klingon Ambassador to the Federation, along with his staff of six aides and a personal cook. The talk among the crew is that the Klingon presence is a consequence of their interest in becoming a profitable part of the rapidly developing Glia starsystem. There is tension in the air, because the Federation is not inclined to allow the Klingons entry at this point, as the Klingons turned down eight years ago an offer to join in the development work, at a time when it looked less profitable. Everyone is expecting the negotiations to be vigorous, but no one is expecting the scenario which is about to unfold.

The Banquet

The negotiations are to start in earnest tomorrow, but tonight is the traditional formal banquet to welcome the ambassador and his staff. While the exchange of pleasantries may well include a few barbed comments, no serious business will be conducted tonight. By great good luck, you have been seated next to Senior Science Officer Spock, and you are counting on Spock to interpret the events of the evening.

Pre-prandial beverages – Gamzain wine, Romulan Ale, Saurian Brandy, and, for the Klingons, chech'tluth – are being served, and the room is alive with the clink of raised glasses and spirited conversation, with the guttural Klingon sounds easily cutting through the general noise. Suddenly the room grows quiet, and you turn to see the Klingon Ambassador standing.

“Heghlu'meH QaQ jajvam,” he rasps. Spock turns to you and translates: “It is a good day to die.” Spock is amused by your shocked expression, and goes on to explain at some length the exalted place held in Klingon culture by the concept of dying with honor.

The Ambassador concludes his brief remarks and is seated, and the general conversation resumes. Servers have begun to bring around appetizers such as Mantickian Pâté and parthas ala Yuta. You notice two Klingons bringing a silver platter to the Ambassador. On the platter is a strange luminous, slightly striated, red-violet object. It appears to be perfectly spherical and between 3 and 4 cm in diameter. You look quizzically at Spock.

“Fascinating,” says Spock. You are seeing something not commonly seen by non-Klingons. That is a serving of the fruit of the K'annTo plant. It is a Klingon delicacy carefully cultivated in hothouses. The fruit is picked when the diameter is precisely 3.4 cm. It has a flavor which appeals only to Klingons. To the Klingons, it is interesting not only for its flavor, but because it is dangerous as well.”

“Dangerous?” you reply. “How can a fruit be dangerous?”

Spock replies, “It has a tiny central core which is toxic. It must be cooked so that the central temperature remains above 80 °C for at least 210 seconds.”

“That sounds easy,” you say. “Why not just boil it for half-an-hour?”

Spock, with the patience of the didact that he is, says “The flavor is destroyed by overcooking. In addition, the Klingons enjoy the sense of danger that comes with minimal cooking, illogical though that is.”

“Now I see why it is important that the diameters have a certain precise value,” you say. “The cooking time is going to depend strongly on the diameter, and one would like to design cookers around a particular size.”

“Impressive,” says Spock with a slight smile. “I see that they are providing good training at the Academy these days. Perhaps after the meal you would like to visit the kitchen and see the cooker, which the Klingons call K’aanTo ‘un.’”

“Yes,” you reply, “I would like that very much.”

Spock continues. “It is a sophisticated bit of electronics. It uses microwaves to add heat uniformly throughout the volume of the fruit and at the same time it uses forced convection to maintain the outer boundary of the fruit at room temperature, thus preventing overcooking of the outer layers. The only control parameter is the time of cooking. Thus the vutwI’ – or as we would say, the cook – must be quite skilled if he is not to decrease the population!”

At that moment you and Spock are startled by a collective gasp from the crowd. You turn to see an alarming sight. The Ambassador is slumped across the silver K’aanTo platter, now empty. His aides are frantically trying to revive him, as Captain Kirk and Doctor McCoy run toward him. Spock jumps up and moves rapidly toward the Ambassador, and you follow.

By the time you reach the group, McCoy has already completed a tricorder scan of the Ambassador, and, with a horrified expression on his face, he says to Kirk, “Jim, he’s dead!”

The silence is broken by C’had, one of the Ambassador’s aides. He turns to Captain Kirk and says in flawless English, “You have poisoned our Ambassador. There can be no peace between our peoples. War is inevitable.”

Kirk looks dumbfounded, but Spock replies immediately to C’had. “You are being impulsive and illogical,” he says. “We do not know the cause of death. It could be many things. Perhaps your Ambassador was in poor health, but we should not guess. We should let the autopsy tell us.”

C’had responds sharply, “The Ambassador was in perfect health, and there will be no autopsy at your barbarian hands.”

Spock raises one eyebrow at being called a barbarian by a Klingon, but he says nothing. Kirk has regained his voice, and, drawing himself up to his full height, says, “As Commander of this ship, I am saying that there WILL be an autopsy. This issue is too important to be left unresolved.”

C’had places one hand menacingly on the dagger in his belt, and says, “NO AUTOPSY. We are returning to our ship with our Ambassador’s body immediately. War between us is inevitable.”

At this point, you are beginning to think that this Cadet Training Mission is getting a bit too realistic. It was all much easier at the Academy where such situations were always hypothetical. Suddenly a thought strikes you, and before you can think better of it, you speak up: “Perhaps it was the K’aanTo fruit.”

Spock, Kirk, McCoy and C'had all stare at you unbelieving – a Cadet speaking in such circumstances?

You turn red but persist. Turning to Spock, you speak again. “Sir, perhaps it was the K’annTo fruit. You know, maybe it wasn’t cooked right.”

C’had snorts in disgust. “Absurd. Klingons always eat K’annTo at ceremonial occasions. They never die from it.”

Spock looks thoughtful and says to C’had, “But suppose the K’annTo were not cooked long enough. Would it not then be lethal?”

C’had protests, “But that never happens. This vutwI’ is very experienced.” C’had turns to one of the other aides and requests that he fetch the vutwI’ from the kitchen.

The aide returns leading a rather frightened looking and quite young Klingon. C’had is surprised and says, “You are not the regular vutwI’. What is the meaning of this?”

The vutwI’ explains he was taken from vutwI’ school when the Ambassador’s regular vutwI’ became very ill just before the trip, and that this was the first time he had ever cooked K’annTo. At this, even C’had begins to look thoughtful.

Again speaking before thinking, you pipe up, “I know how to calculate temperature variations in heated spherical solids. I can calculate the cooking time if the vutwI’ will tell me how long the power was on in the K’annTo ‘un.’”

Spock can’t quite repress a smile, and he asks you just how you would go about that calculation. You explain in great detail, and as you talk the smile disappears. You finish your explanation, and there is a brief silence.

Spock turns to Kirk and says, “Captain, this Cadet is well-versed in thermal partial differential science. I believe that he can indeed establish with his calculations whether the K’annTo fruit was properly cooked. It is a calculation which might avert a war. I strongly urge you to let him try it.”

Kirk thinks briefly, and then says, “All right. But it must be carried out with dispatch.” He then turns to C’had and says, “Will you accept the results of such an analysis?”

C’had protests, “But he is one of you. How can I trust what he calculates?”

Spock replies, “You can check his work.” At this, C’had looks a little uncomfortable, as he is much more at home with dagger and shield than the sharp-edged partial differential equations of heat transfer.

Finally C’had replies, “If the calculations show that the K’annTo was not properly cooked, then I will forward them to Qo’noS for verification. If the calculations show that the K’annTo was properly cooked, then we shall depart immediately and war is inevitable.”

Although Spock has already pointed out that there are many possible causes of death other than deliberate poisoning, he says nothing further because he realizes that the Klingons will give no more ground. Spock is a student of Klingon culture and he knows that to kill by poisoning is the worst possible and most dishonorable crime to them, because one kills one’s enemy without facing that enemy.

Another Cadet – your traveling partner from the Academy – joins the group, and you introduce her to the officers, telling them that her mathematical skills are at least the equal of your own. She turns to the vutwI' and asks the crucial question: “How long was the power on when you cooked the K’annTo?”

The young vutwI', managing to look both defiant and scared, replies in heavily accented English, “325 seconds. Plenty of time. Plenty of time.”

At this point, you all head for the kitchen to retrieve the K’annTo ‘un. Mr. Scott will do a few measurements to verify that the power level is up to spec. Meanwhile the two of you need to begin the theoretical calculations. The Klingons are not noted for their patience, and a quick result would be good. You head for the MacTrek Lab on the Hopemen Deck with the future of peace in the Federation in your hands.

The officers watch the two of you go. Kirk is relieved that the whole mess is in someone else’s hands. McCoy is a little disappointed at not being able to do the Klingon autopsy. C’had is still angry that two Cadets have delayed a glorious, honorable and inevitable war. Spock is wondering about the logic of leaving the responsibility for peace in in the hands of two young and unseasoned Cadets.

The Cooking Equations

The temperature in the cooking K’annTo fruit is a function only of the spherical radial coordinate r and the time t . The equation governing the temperature is the heat equation with a source term:

$$\rho c \frac{\partial T}{\partial t} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + \Gamma, \quad (\text{power on, } 0 < t < \tau) \quad (1)$$

where ρ is the density, c is the specific heat, k is the thermal conductivity, and Γ is the rate of heat addition per unit volume. This equation applies as long as the power is on. When the power is off, the equation is

$$\rho c \frac{\partial T}{\partial t} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right). \quad (\text{power off, } \tau < t) \quad (2)$$

The power is turned on at time $t = 0$, and then turned off at time $t = \tau$. The cook chooses the value of τ , and the choice is crucial for the correct cooking. At the initial time, the fruit is at room temperature T_0 , and at all times the outer boundary ($r = a$) is maintained at room temperature T_0 . These conditions are

$$T(r,0) = T_0, \text{ and } T(a,t) = T_0. \quad (3)$$

The above three equations define completely the solution $T(r, t)$. After T is found, the central temperature $T(0,t)$ must be examined. For proper cooking, this central temperature must exceed a prescribed value T_c for a minimum time t_c . We give next the values of all of these parameters.

The Parameter Values

We give all of the relevant parameter values below, along with the thermal diffusivity D , which is defined by

$$D = \frac{k}{\rho c}.$$

$$\rho = \text{density} = 1020.0 \text{ kg/m}^3$$

$$c = \text{specific heat} = 3980.0 \text{ J/kg } ^\circ\text{C}$$

$$k = \text{thermal conductivity} = 0.65 \text{ W/m } ^\circ\text{C}$$

$$\Gamma = \text{energy addition rate} = 1.3 \times 10^6 \text{ W/m}^3 \text{ (measured by Mr. Scott)}$$

$$a = \text{sphere radius} = 0.017 \text{ m}$$

$$T_0 = \text{initial and boundary temperature} = 20.0 \text{ } ^\circ\text{C}$$

$$T_c = \text{minimum required cooking temperature} = 80.0 \text{ } ^\circ\text{C}$$

$$t_c = \text{minimum cooking time} = 210.0 \text{ s}$$

$$\tau = \text{time power is on} = 325.0 \text{ s}$$

$$D = k/\rho c = \text{thermal diffusivity} = 1.60 \times 10^{-7} \text{ m}^2/\text{s}$$

The value $\tau = 325 \text{ s}$ is the value used by the cook. It is the only adjustable parameter in the cooking process, and our task is to determine whether it was an adequate value for safe cooking.

Useful Information on Solution Technique

The problem specified by equations (1) through (3) becomes much simpler if you make the following change to a new dependent variable $\Psi(r, t)$:

$$\Psi(r, t) = r(T(r, t) - T_0). \quad (4)$$

Notice that this implies a condition on Ψ at $r = 0$, because T must be well-behaved there. Once you have reformulated the problem in terms of Ψ , you should look for a solution in the form of an expansion in an appropriate set of r -eigenfunctions. You will have to solve equation (1) for $t < \tau$ and equation (2) for $t > \tau$. The solution defined in these two time ranges has to be continuous at the transition $t = \tau$.

Work Schedule

In order to allow time for the evaluation of your work before term-end, your report on the K'aanTo cooking must be completed by stardate 9622.3 (local Rochester time: Wednesday December 13 by 6 PM EDT). Your promotion to the next class at Starfleet Academy, to say nothing of the preservation of the peace between the Federation and the Klingons, depends on the correctness of your work.

FIGURES



Figure 1. K'aanTo Fruit. A Klingon delicacy used as an appetizer at formal banquets. The core is toxic unless a rather exacting cooking procedure is followed.



Figure 2. K'aanTo 'un. The sophisticated electronic cooker used by the Klingon cooks for K'aanTo. The interior is heated uniformly by microwaves and the surface is maintained at room temperature by forced convection.

Cover picture from www.startrek.com, copyright Paramount Pictures. Figure 1 courtesy of Prof. P. A. Clark of RIT. Figure 2 from NASA web site. Help with the Klingon Language from **Star Trek Klingon for the Galactic Traveler**, Marc Okrand, Pocket Books, 1997.