ME 201 / MTH 281Fall 2009
Integrals Useful in Fourier Analysis

This document summarizes the integrals which were useful in our calculation of the coefficients in Fourier series. The important concept behind these results is orthogonality, which we will encounter in a more general context when we study Sturm-Liouville systems. We present the results as a table. By way of example we give a derivation of one of the results. In all of the formulas below, *m* and *n* are positive integers.

$$\int_{-L}^{L} \sin(n\pi x/L)\,dx = 0, \text{ and } \int_{-L}^{L} \cos(n\pi x/L)\,dx = 0.$$

$$\int_{-L}^{L} \sin(n\pi x/L)\cos(n\pi x/L)\,dx = 0.$$

$$\int_{-L}^{L} \cos(n\pi x/L)\cos(m\pi x/L)\,dx = \begin{Bmatrix} 0, & m \neq n \\ L, & m = n \end{Bmatrix}.$$

$$\int_{-L}^{L} \sin(n\pi x/L)\sin(m\pi x/L)\,dx = \begin{Bmatrix} 0, & m \neq n \\ L, & m = n \end{Bmatrix}.$$

By way of example, we derive the last result in the table. We start with the trig identity

$$\sin(\theta)\sin(\phi) = \tfrac{1}{2}\left\{\cos(\theta-\phi) - \cos(\theta+\phi)\right\}.$$

Then the integral in the last table entry may be written as $I_1 - I_2$, where

$$I_1 = \tfrac{1}{2}\int_{-L}^{L} \cos\left[(n-m)(\pi x/L)\right]dx, \text{ and } I_2 = \tfrac{1}{2}\int_{-L}^{L} \cos\left[(n+m)(\pi x/L)\right]dx.$$

We do I_2 first: $I_2 = \dfrac{1}{2}\dfrac{L}{(n+m)\pi}\sin\left[(n+m)\dfrac{\pi x}{L}\right]_{-L}^{L} = \dfrac{L}{(n+m)\pi}\sin[(n+m)\pi] = 0.$

Now I_1. For $m \neq n$,

$$I_1 = \frac{1}{2}\frac{L}{(n-m)\pi}\sin\left[(n-m)\frac{\pi x}{L}\right]_{-L}^{L} = \frac{L}{(n-m)\pi}\sin[(n-m)\pi] = 0.$$

For $m = n$, $I_1 = \tfrac{1}{2}\int_{-L}^{L} 1\,dx = L.$