ME 201/MTH 281 EXAM #1

THURSDAY OCTOBER 9, 2008 2:00 – 3:15 PM and 3:25 – 4:40 PM

This exam covers assignments 1 through 4, and Chapters 1 and 2 and section 3.1 of the class notes. You may use any books, notes or other references, but you may not exchange material with anyone else. If you need additional information to work a problem, ask me for it, and, if it is appropriate, I will put it on the board. Do all four problems. **Be sure to explain your work! Wrong calculations with no explanation will receive very little partial credit.** Solutions will be posted on the web when both exams are over, and printed solutions will be distributed in class on Friday Oct. 10. Graded exams will be returned in class on Monday October 13. Exam statistics will be posted on the web. Good luck!

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(1) (25 points) Consider the modified one-dimensional heat conduction problem given below in which U, D, γ, β and T_0 are all positive constants. Look for separated solutions of the form $F(x)G(t)$. Find the equations satisfied by F and G, and the boundary conditions satisfied by F. Do NOT try to solve the equations for F and G.

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} = D\frac{\partial^2 T}{\partial x^2} - \gamma e^{-\beta t}T, \quad 0 < x < L,$$

$$\text{with } \frac{\partial T}{\partial x}(0,t) = 0, \quad T(L,t) = 0, \text{ and } T(x,0) = T_0(x/L).$$

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(2) (25 points) Consider the function $f(x) = 2x - x^2$. For each series below, sketch three periods of the periodic function represented by the series, and tell how rapidly the Fourier coefficients a_n and/or b_n will decrease with n. You do NOT need to calculate any Fourier coefficients to answer these questions.

(a) Full Fourier series of $f(x)$ on $-1 \le x \le 1$.

(b) Fourier cosine series of $f(x)$ on $0 \le x \le 1$.

(c) Fourier sine series of $f(x)$ on $0 \le x \le 1$.

(d) Fourier sine series of $f(x)$ on $0 \le x \le 2$.

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(3) (35 points) Solve the initial value problem given below, in which D and T_0 are positive constants.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} \ ,\ 0 < x < L \ ,\ t > 0 \ ,$$
$$\text{with } T(0,t) = 0 \ ,\ T(L,t) = T_0 \ ,$$
$$\text{and } T(x,0) = T_0\left[\frac{x}{L} + \sin\left(\frac{\pi x}{L}\right) - \sin\left(\frac{3\pi x}{L}\right)\right].$$

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(4) (15 points) You are the CEO of the Phoenix Fire Wall Company, makers of leaner, meaner and more prestigious firewalls. Your chief engineer is excitedly telling you about a new refractory material for firewalls. He claims that a 0.5 meter thickness of this material will provide fire protection for 10 hours, and that an economy version, which is 0.25 meters thick, will provide fire protection for 5 hours. Do you give him a raise or fire him? Explain your answer.
