ME 201/MTH 281 ASSIGNMENT #9 2008

Assignments handed in by 6 PM on Wednesday Nov. 19 will receive a 5 point bonus. Assignments handed in after that but by 6 PM on Thursday Nov. 20 will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday Nov. 20. Note that problem 4 is a 15-point bonus problem.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
VI. UNBOUNDED DOMAINS		
6.3 Laplace Equation	W,Th Nov 5,6	10.6.3
6.4 Similarity Method	F Nov 7	---
6.5 Kelvin's Estimate of the Earth's Age	M Nov 10	---
Exam Review	W Nov 12	---
Exam #2	Th Nov 13	
VII. PROBLEMS IN SPHERICAL COORDINATES		
7.1 Review of Power Series	F Nov 14	---
7.2 Power Series Solutions of Ordinary Differential Equations	M Nov 17	---

PROBLEMS

(1) (15 points) In class you were given the following Fourier transform: $\mathscr{F}\{e^{-x^2}\} = \sqrt{\pi}\, e^{-k^2/4}$. Use this and the derivative rule to show that $\mathscr{F}\{x^2\, e^{-x^2}\} = (\sqrt{\pi}\,/\,4)(2-k^2)e^{-k^2/4}$**.** You will use this transform in problem (2).

<u>Laplace Equation in an Infinite Strip</u>

(2) (40 points) Consider the boundary value problem given below for the electrostatic potential $\Phi(x,y)$ in an infinite strip. The boundaries of the strip are $y=0$ and $y=b$, and the strip extends from $-\infty$ to $+\infty$ in x. The potentials on the boundaries of the strip are specified. The complete formulation of the problem for Φ in is given below.

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0\ ,\ -\infty<x<\infty\ ,0<y<b\ ,$$

$$\text{with } \Phi(x,0)=0, \text{ and } \Phi(x,b)=\Phi_0\left(\frac{x}{b}\right)^2 e^{-(x/b)^2}\ .$$

Here Φ_0 is a positive constant. We also require that $\Phi\to 0$ as $x\to\pm\infty$.

(a) (10 points)Put the problem in dimensionless form by introducing the following dimensionless variables:

$$\hat{x}=x/b,\ \hat{y}=y/b, \text{ and } \hat{\Phi}=\Phi/\Phi_0.$$

In the remainder of the problem you may drop the hats on the scaled variables.

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(b) (10 points) Use the Fourier transform in x to obtain an explicit solution for $\tilde{\Phi}(k,y)$, the Fourier transform of $\Phi(x,y)$ with respect to x. Write down the inversion integral which gives $\Phi(x,y)$ in terms of $\tilde{\Phi}(k,y)$.

(c) (10 points) Using the inversion integral of part (b), show that the potential on the line $x = 0$ can be written as

$$\Phi(0,y) = \frac{1}{4\sqrt{\pi}} \int_0^\infty (2-k^2) e^{-k^2/4} \frac{\sinh(ky)}{\sinh(k)} dk \ .$$

(d) (10 points) Use Nintegrate in Mathematica to calculate values of $\Phi(0,y)$. Plot this function from $y = 0$ to $y = 1$. Use your plot to estimate the maximum value of $\Phi(0,y)$ for y in this range.

Similarity Problems

(3) (30 points) Solve the problem below by a similarity method. Be sure to explain your similarity arguments in detail. You should obtain an explicit solution in terms of the error function. The calculation will be very similar to the one in section 6.4 of the class notes. The temperature T_0 is constant, and the diffusivity D is a positive constant.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0, \text{ with } T(x,0) = T_0 \text{ for } x < 0, \text{ and } T(x,0) = 0 \text{ for } x \geq 0 .$$

Bonus Problem

(4) (15 points)This problem was first discussed by L. I. Sedov and G. I. Taylor in the 1940's. Sedov is the author of a classical book on similarity methods: **Similarity and Dimensional Methods in Mechanics**, L. I. Sedov, Academic Press 1959.

An intense spherically symmetric explosion is produced in air. The energy released in the explosion is E. The density of the air before the explosion is ρ. The explosion is so intense that we may neglect the pressure in the air before the explosion. We also neglect gravity and the buoyancy of the air heated by the explosion. Assume that the explosion starts at a point at time t = 0 and then expands spherically. Use dimensional analysis to predict how the radius R of the sphere will increase with time.

Power Series Solution of Ordinary Differential Equations

(5) (15 points) Solve each of the differential equations given below for $y(x)$ with a power series expansion. Unless otherwise stated, the expansion is to be taken about $x = 0$. In each case, find the first four nonzero terms in the series.

(a) (5 points) $y'' + \frac{y}{1+x} = 0$, $y(0) = 1$, $y'(0) = -1$.

(b) (5 points) $y'' + 4y = 0$, $y(0) = 0$, $y'(0) = -2$.

(c) (5 points) $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$, $y(1) = 1$, $y'(1) = 2$, power series expansion about $x = 1$.