ME 201/MTH 281 ASSIGNMENT #7 2008

Revised due dates. Assignments handed in by 6 PM on Thursday Oct. 30 will receive a 5 point bonus. Assignments handed in after that but by 6 PM on Friday Oct. 31 will receive full credit but no bonus. No assignments will be accepted after 6 PM on Friday.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
V. SEPARATION OF VARIABLES, PART 2		
5.1 Heat Conduction with Newton's Law of Cooling	Th Oct 23	5.8
5.2 Heat Conduction with Sources	F Oct 24	8.3
5.3 Critical Size of a Nuclear Reactor	M Oct 27	---

PROBLEMS

(1) (40 points) The object of this assignment is to make use of the Mathematica notebook entitled "Newton's Law of Cooling" to solve a heat conduction problem very similar to the one solved in class in section 5.1 of the notes. The notebook will be handed out in class on Thursday Oct. 23, and is available on the web now. Your task is to modify that notebook so that it solves the following problem:

$$\frac{\partial T}{\partial t} = D_f \frac{\partial^2 T}{\partial x^2}, 0 < x < L, t > 0,$$

$$\text{with } \frac{\partial T}{\partial x}(0,t) = 0,\ k\frac{\partial T}{\partial x}(L,t) + h[T(L,t) - T_A] = 0,$$

$$\text{and } T(x,0) = T_L + (T_R - T_L)\frac{x}{L}.$$

The values of the constants and parameters in the problem are

$$h = 22.4 \text{ W/m}^2\cdot\text{K},$$

$$L = 0.5 \text{ m},$$

$$k = 2.80 \text{ W/m}\cdot\text{K},$$

$$D_f = 1.37\times 10^{-6} \text{ m}^2/\text{s},$$

$$T_L = -30^\circ\text{C}, T_A = 10^\circ\text{C}, T_R = 50^\circ\text{C}.$$

(a) (10 points) Find the steady-state solution. Show that the average initial temperature is equal to the final steady-state temperature, and hence that the final energy in the slab is the same as the initial energy. (This is a parametric accident for the numbers given here, and not a general result for this problem.)

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(b) (15 points) Modify the notebook "Newton's Law of Cooling" to solve the above problem. Find and use the first 100 terms in the series rather than the first 40.

(c) (15 points) In this part of the problem, you will use your solution to learn something about the heat flux leaving the slab at the face $x = L$. Use your series solution to get a graph of the heat flux versus time from the initial time to 100 hours. Find the time at which the heat flux reverses direction. Give a simple theoretical argument which shows why you would get zero if you were to integrate this heat flux with respect to time from 0 to 100 hours.

((2) (30 points) Solve the boundary value problem below which describes the transient heat flow in a slab of width L produced by a heat source γ which depends on x and t. (Connection with our earlier notation: If Γ is the rate of energy addition per unit volume, then $\gamma = \Gamma / \rho C$, where ρ is the density and C is the specific heat.)

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + \gamma(x,t), \text{where } \gamma(x,t) = T_0 \alpha (x/L) e^{-\alpha t} \ ,$$

$$\text{and where } T(0,t) = 0, T(L,t) = 0, \text{and } T(x,0) = 0 \ .$$

Here α and T_0 are positive constants. Hint: Look for a solution in the form

$$T(x,t) = \sum_{n=1}^{\infty} C_n(t) \sin\left(\frac{n\pi x}{L}\right).$$

((3) (30 points) A nuclear reactor is made in the shape of a rectangular parallelepiped occupying the region $0 \le x \le a, 0 \le y \le b, \text{and } 0 \le z \le c$. The reactor core design is such that the neutron diffusivity is different in each of the three coordinate directions. The governing equation for the neutron density is then

$$\frac{\partial N}{\partial t} = D_x \frac{\partial^2 N}{\partial x^2} + D_y \frac{\partial^2 N}{\partial y^2} + D_z \frac{\partial^2 N}{\partial z^2} + \alpha N - \beta N \ .$$

Here the three diffusivities $D_x, D_y, \text{and } D_z$ are positive constants. As usual, α and β are positive, with $\alpha > \beta$. The boundary condition is that the neutron density N should vanish on the edges of the rectangular parallelepiped. The value of the height c is fixed, whereas a and b are to be determined so that the reactor has minimum volume. Find those values of a and b. Recall from calculus that a convenient way to find the extremum of a function when there is a constraint is the method of Lagrange multipliers.