ME 201/MTH 281 ASSIGNMENT #6 2008

Assignments handed in by 6 PM on Thursday Oct. 23 will receive a 5 point bonus. Assignments handed in after that but by 4 PM on Friday Oct. 24 will receive full credit but no bonus. No assignments will be accepted after 4 PM on Oct. 24.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
3. Separation of Variables, Part I		
3.6 Beyond Fourier Series	Th Oct 16	---
4. Sturm Liouville Theory		
4.1 Eigenfunctions and Eigenvalues	F, M Oct 17, 20	5.1-5.3, 5.5
4.2 Eigenfunction Expansions	W Oct 22	5.3

PROBLEMS

(1) (60 points) Consider the regular Sturm-Liouville problem given below.

$$\frac{d}{dx}\left(x\frac{d\psi}{dx}\right)=-\frac{\lambda\psi}{x} \quad , \quad 1<x<2, \text{ with } \psi(1)=0 \text{ and } \psi(2)=0 \quad .$$

(a) (10 points) Prove that the eigenvalues are positive.

(b) (15 points) From your work in MTH 163 or MTH 165, you should recognize the equation as an equidimensional equation. Such equations have solutions of the form x^r , where the possible values of *r* are determined by substituting the form into the equation. Recall, again from 163 or 165, that for any real constant α, $x^{i\alpha}=\cos(\alpha\ln[x])+i\sin(\alpha\ln[x])$. Use this information to find the general solution. Then show that the eigenvalues and eigenfunctions of this system are given by

$$\lambda_n=\frac{(n\pi)^2}{(\ln 2)^2} \quad , \quad \psi_n=\sin\left(\frac{n\pi\ln(x)}{\ln(2)}\right) \quad , \quad n=1,2,3,\ldots$$

(c) (10 points) Verify that the eigenfunctions satisfy the appropriate orthogonality relation.
(d) (15 points) Use Mathematica to plot the first five eigenfunctions. Verify for these five that the n^{th} eigenfunction has *n*-1 zeros in the interior of the interval.
(e) (10 points) Obtain explicitly the expansion in these eigenfunctions of $f(x)=\ln(x)/\ln(2)$. Use Mathematica to plot your series, and show on the same plot the function $f(x)$.

(2) (40 points) Consider the boundary value problem

$$\frac{1}{x}\frac{\partial\Phi}{\partial t}=\frac{\partial}{\partial x}\left(x\frac{\partial\Phi}{\partial x}\right)-1 \quad , \quad 1<x<2 \; , \; t>0 \; ,$$

with $\Phi(1,t)=1$, $\Phi(2,t)=1$, and $\Phi(x,0)=x$.

(a) (30 points) Use your results of problem (1) and separation of variables to solve the problem.

(b) (10 points) Find a simple approximation for large times consisting of the steady-state solution and the first term in the separation of variables solution for the transient. Estimate the time range for which your approximation is valid.