ME 201/MTH 281 ASSIGNMENT #4 2008

Assignments handed in by 6 PM on Wednesday Oct. 1 will receive a 5 point bonus. Assignments handed in after that but by 6 PM on Thursday will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday Oct. 2.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
2. Fourier Series		
2.4 Fourier Sine and Cosine Series	W Sept. 24	3.3
2.5 Separation of Variables Revisited	Th Sept. 25	Chapter 2
3. Separation of Variables, Part I		
3.1 Diffusion Equation	F, M Sept. 26, 29	8.2

PROBLEMS

2.4 FOURIER SINE AND COSINE SERIES

(1) (20 points) In each of the problems below, find explicitly the series asked for, sketch the periodically extended function represented by the series, and show that the rate of convergence of the series is consistent with the smoothness of the periodically extended function. Tell what the series converges to at each point of the given interval.

(a) The full Fourier series for $f(x) = \sin(\pi x)$ on $-2 \le x \le 2$.

(b) The Fourier cosine series for $f(x) = \sin(\pi x)$ on $0 \le x \le 2$.

(c) The Fourier sine series for $f(x) = x(1-x)$ on $0 \le x \le 1$.

(d) Both the Fourier sine and cosine series for $f(x) = 2$ on $0 \le x \le 1$

2.5 SEPARATION OF VARIABLES REVISTED
3.1 DIFFUSION EQUATION

(2) (15 points) Spherical potatoes of varying mass *m* are being cooked by dropping them in boiling water. Assume that the boiling produces sufficient stirring to maintain a constant temperature (that of boiling water) on the surface of each potato. Using the basic scale relation of diffusion, show that the cooking time is proportional to m^{α} . Your analysis should give you a specific value for the exponent α. State any assumptions you make.

(CONTINUED NEXT PAGE)

(3) (65 points) Consider the initial value problem given below for $T(x,t)$.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} + \beta x \quad , \quad 0 < x < L \ , t > 0 \ , T(0,t) = 0 \ , T(L,t) = 0 \ , T(x,0) = T_0 \ ,$$

where β and T_0 are constants. This is the equation for transient heat conduction in a bar of length *L,* with the ends maintained at zero temperature, and with a heat source of strength $\Gamma = \rho c \beta x$.

(a) (10 points) Follow the procedure given in class, and split the solution *T* into a steady-state part T_s and a transient part $\hat{T}$. Give a complete formulation (equation, boundary conditions, and, for the transient, an initial condition) for the determination of T_s and $\hat{T}$.

(b) (10 points) Find the steady-state solution T_s. You may make use of any results derived in class.

(c) (10 points) Find the transient solution $\hat{T}$. You may make use of any results obtained in class, but be sure to explain your work.

(d) (5 points) Verify that your solution *T* approaches the steady-state solution as time *t* goes to infinity. On the basis of this observation, plus any others you might want to make about the solution, would you agree or disagree with the following somewhat vague statement?

Solutions of the diffusion equation remember their initial conditions.

(e) (10 points) Sometimes we want to have more information than the simple result of (b) gives us. Show that for large times, the series solution for the transient $\hat{T}$ reduces to a simple one-term approximation. Estimate how large the time must be for this approximation to be valid.

(f) (10 points) For the following parameter values, estimate the time at which the temperature and the steady-state temperature differ by less than $\pm 2^\circ\text{C}$ throughout the bar: $T_0 = 30^\circ\text{C}$, L = 0.1 m, $\beta = 480\ ^\circ\text{C}/(\text{m} \cdot \text{s})$, and $D = 2.0 \times 10^{-4}\ \text{m}^2/\text{s}$.

(g) (10 points) For the parameter values given in (f), use Mathematica to construct some graphs of *T* versus *x* for various values of time *t*. The two principal practical problems you must solve to make this useful are (1) the determination of the number of terms in the series needed for reasonable accuracy, and (2) the determination of a set of time values for the graphs so that you get a good overview of the entire diffusion process.