ME 201/MTH 281 ASSIGNMENT #2 2008

Assignments handed in by 6 PM on Wednesday September 17 will receive a 5 point bonus. Assignments handed in after that but by 6 PM on Thursday September 18 will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
1.3 Separation of Variables – A First Attempt	M,W Sept. 8,10	2.1, 2.2, 2.3.1-2.3.5
2.1 Basic Fourier Series	Th,F,M Sept. 11,12,15	3.1

PROBLEMS

1.3 SEPARATION OF VARIABLES

(1) (20 points) Find an explicit solution to the boundary value problem given below.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} \quad , \quad 0 \le x \le L \ , t \ge 0 \ , \quad T(0,t) = 0 \ , T(L,t) = 0 \ , T(x,0) = T_0 \sin\left(\frac{\pi x}{L}\right) + T_1 \sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right),$$

where T_0 and T_1 are constants. (Hint: You will need to use some standard trig identities to transform the initial condition.)

(2) (30 points) For the boundary value problem given below for heat conduction in a bar, carry out a separation of variables solution, and take the calculation as far as you can with what we have learned so far. In particular, you should find all of the separated solutions that satisfy the boundary conditions and then attempt to superpose them to satisfy the initial conditions. Your calculation here will closely resemble the one presented in class in section 1.3 of the class notes.

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2} \ , 0 \le x \le L \ , t \ge 0 \ , T(0,t) = 0 \ , \frac{\partial T}{\partial x}(L,t) = 0 \ , T(x,0) = T_0 \ .$$

This corresponds to the right end of the bar being insulated, the left end being maintained at zero temperature, and an initial constant temperature.

2.1 BASIC FOURIER SERIES

(3) (25 points) Consider the function $f(x)$ defined by

$$f(x) = x + |x| \text{ for } -1 \le x < 1, \text{ and } f(x+2) = f(x) \text{ for all } x.$$

(a) (5 points) Give a sketch of $f(x)$ versus x for $-2 \le x \le 2$.

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(3) (continued)

(b) (5 points) Show that *f* is piecewise smooth.

(c) (10 points) Find the Fourier series for *f*.

(d) (5 points) Using the basic convergence theorem, tell what the series converges to for each *x* in the interval [-1,1].

COMPUTER WORK

For this assignment, you are asked to work through the following five sections of the Mathematica Tutorial: Arithmetic; Algebra; Defining Constants, Expressions and Functions; Plotting; Summing Series. You may download the tutorial from the course web site. After you have loaded the tutorial notebook into Mathematica, you can open any of the sections by double clicking on the square bracket opposite the section name. You do not need to hand in any of your work with the tutorial.

(4) (25 points) In this problem, you will continue working with the series of problem 3.

(a) (5 points) Use Mathematica to graph the function *f* over the interval [-2,2]. (Hint: If you have defined a function fbas[x] over the interval [-*L*, *L*], the extension of fbas which is periodic with period 2*L* can be defined in Mathematica by f[x_] := fbas[Mod[x, 2*L, -L] .)

(b) (10 points) Use the Sum function, which you worked with in the tutorial, to define a function which plots the partial sum based on the first M terms of the series. By Mth partial sum, the following is meant:

$$a_0 + \sum_{n=1}^{M} \{a_n \cos(n\pi x) + b_n \sin(n\pi x)\} .$$

Your function should take *M* as an argument, and produce a graph of the above expression over the *x*-range [-2,2].

(c) (10 points) Use your function of part (b) to produce three plots of partial sums for *x* in the interval [-2,2]: M = 1, M = 5, and M = 10.