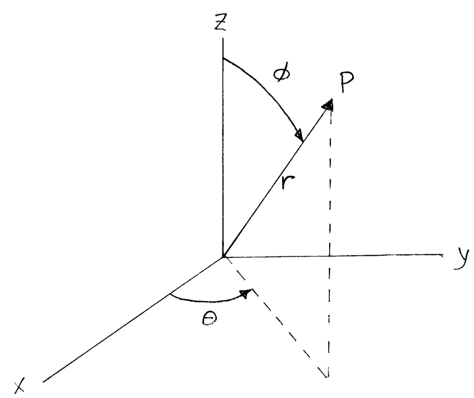
ME 201/MTH 281 ASSIGNMENT #10 2008

This is the last required assignment. There will be an additional practice assignment covering cylindrical coordinates. Assignments handed in by 6 PM on Wednesday December 3 will receive a 5 point bonus. Assignments handed in after that but by 6 PM on Thursday December 4 will receive full credit but no bonus. No assignments will be accepted after 6 PM on December 4.

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
VII. SPHERICAL COORDINATES		
7.3 Separation of Variables in Spherical Coordinates	W Nov 19	7.10
7.4 Legendre's Equation and Legendre Polynomials	Th,F Nov 22, 23	7.10
7.5 Laplace Equation in a Sphere	M Nov 24	7.10
7.6 Poisson Equation in a Sphere (Project Problem)	W Nov 26	7.10
THANKSGIVING BREAK	Th, F Nov 27, 28	



Spherical Coordinates Used in These Problems

PROBLEMS

(1) (20 points) Solve the boundary value problem given below for the axisymmetric electrostatic potential $\Phi(r,\phi)$ inside a sphere of radius a. The quantity ! $_0$ is a constant.

$$!^{2} \quad = 0\ , r < a \text{ and } 0 \ \# \quad \# \ \%, \text{ with } \quad (a, \ ) = \quad {}_0\cos^3(\ ) .$$

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(2) (30 points) Solve the boundary value problem given below for the axisymmetric electrostatic potential between concentric spheres of radii a and b. The inner sphere of radius a is a conductor, and has a constant potential V. The potential is a specified function of ϕ on the surface of the outer sphere of radius b. The problem formulation is given below. The quantities V, V_1 and V_2 are all constants.

$$\nabla^2\Phi = 0 \text{ , } a < r < b \text{ and } 0 \le \phi \le \pi, \text{ with } \Phi(a,\phi) = V \text{ and } \Phi(b,\phi) = V_1 + V_2 \cos^2\phi \text{ .}$$

(3) (50 points) In this problem you will solve for the potential flow of a fluid past a rigid sphere. You can pattern the calculation closely after the one presented in class for a conducting sphere in an applied electric field (section 7.5 of the notes).

A fluid flow is called a potential flow if the velocity $\mathbf{V}$ may be expressed as the gradient of a potential: $\mathbf{V} = \nabla\Phi$. It can be shown that for an incompressible potential flow, Φ satisfies $\nabla^2\Phi = 0$, and this is the equation you will solve in this problem. For a uniform flow with velocity V_0 in the positive z-direction, the potential is $\Phi_0 = V_0 z$ – you can easily check that $\nabla\Phi = V_0\mathbf{k}$. The sphere will disturb this uniform flow, and the velocity potential with the sphere present is $\Phi = \tilde{\Phi} + \Phi_0$, where $\tilde{\Phi}$ represents the disturbance due to the sphere. The boundary condition on the surface of the sphere is that no fluid can flow into the sphere, hence on the surface of the sphere $\mathbf{V}\cdot\mathbf{n} = 0$, where $\mathbf{n}$ is the unit normal to the sphere. Because the normal to the sphere is in the r-direction, this condition may be expressed as $\left.\frac{\partial\Phi}{\partial r}\right|_{r=a} = 0$. A complete formulation of the problem for $\tilde{\Phi}$ is given below.

$$\tilde{\Phi} = \tilde{\Phi}(r,\phi) \text{ , } \nabla^2\tilde{\Phi} = 0 \text{ for } r > a \text{ , } 0 \le \phi \le \pi \text{ ,}$$

$$\text{with } \frac{\partial\tilde{\Phi}}{\partial r} = -\frac{\partial\Phi_0}{\partial r} \text{ on } r = a \text{ , and } \tilde{\Phi} \underset{r\to\infty}{\longrightarrow} 0 \text{ .}$$

The last condition, $\tilde{\Phi} \underset{r\to\infty}{\longrightarrow} 0$, says that the disturbance created by the sphere becomes vanishingly small as we move far away from the sphere.

(a) (25 points) Solve the boundary value problem for $\tilde{\Phi}$.

(b) (10 points) The disturbance to the uniform velocity produced by the sphere goes to zero as one moves far from the sphere. Use your solution for $\tilde{\Phi}$ to determine how rapidly the disturbance velocity decays with r, the distance from the center of the sphere.

(c) (10 points) Use your solution to find the speed of the fluid on the surface $r = a$ of the rigid sphere. What is the maximum speed of the fluid and where does it occur?

(d) (5 points) You are a passenger in a car moving at a constant speed of 60 mi/hr. You open the right side window and hold a velocity-measuring device out the window (a pitot tube or a rotating cup anemometer). Will the speed measured by the device be (a) less than 60 mi/hr, (b) approximately equal to 60 mi/hr, or (c) greater than 60 mi/hr? Explain your answer.