ME 201/MTH 281 ASSIGNMENT #1 2008 Revised

Assignments handed in by 6 PM on Wednesday September 10 will receive a 5-point bonus. Assignments handed in after that but by 6 PM on Thursday September 11 will receive full credit but no bonus. No assignments will be accepted after 6 PM on Thursday. Most of this assignment is review material from MTH 163 (or MTH 165) and MTH 164. Note the 5-point bonus problem in the second part of **4 (a).**

LECTURE SCHEDULE AND READING

Section in Class Notes	Date	Section in Text
1.1 Continuous Systems and Partial Differential Equations	W Sept. 3	1.1
1.2 Heat Conduction and the Diffusion Equation	Th,F Sept. 4,5	1.2-1.5

REVIEW PROBLEMS FROM MTH 163 AND 164

Vector Calculus

In problems 1-4 below, we use a standard right-handed rectangular coordinate system, with coordinates x, y, and z, and with unit vectors **i**, **j**, and **k**. In these problems $F(x,y,z) = x^2 + y^2 - 2z^2$, and $\mathbf{H}(x,y,z) = yz\,\mathbf{i} - xz\,\mathbf{j} + z\,\mathbf{k}$.

(1) (10 points) Calculate the following: ∇F, $\nabla \cdot \mathbf{H}$, $\nabla \cdot (F\mathbf{H})$, $\nabla \times \mathbf{H}$.

(2) (10 points) Suppose the temperature distribution in three-dimensional space is given by the function AF, where x, y, and z are in meters, and where A = 1 °C/m^2 . At the point $x = 1$, $y = 2$, and $z = 2$, find a unit vector in the direction of the maximum rate of increase of temperature. What is this maximum value?

(3) (10 points) Let S be the surface of the ellipsoid $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$. Here a, b, and c are positive constants. Let **n** be the unit exterior normal to the surface. Show that $\oiint_S \mathbf{H} \cdot \mathbf{n}\, d\sigma = \frac{4}{3}\pi abc.$

(CONTINUED NEXT PAGE)

(4) (10 points) Let C be the circle $x^2 + y^2 = 1$ in the plane $z = 0$. In the integrals below, the circle is traversed counterclockwise when viewed from above the plane $z = 0$.

(a) Show that $\oint_C \mathbf{H} \cdot d\mathbf{s} = 0$. For a 5-point bonus, use Stokes' Theorem to evaluate $\oint_D \mathbf{H} \cdot d\mathbf{s}$ where D is the circle $x^2 + y^2 = 1$ in the plane $z = 1$, with the circle traversed counterclockwise when viewed from above the plane $z = 1$.

(b) Show that $\oint_C \nabla F \cdot d\mathbf{s} = 0$.

<u>Differential Equations</u>

(5) (10 points) Solve the given initial-value problem. $\dfrac{dx}{dt} + 2x = e^t$, $x(0) = 0$.

(6) (10 points) Solve the given initial-value problem. $\dfrac{dx}{dt} + 2x = e^{-2t}$, $x(0) = 0$.

(7) (10 points) Solve the given initial-value problem. $\dfrac{d^2y}{dx^2} + 9y = 0$, $y(0) = 1$, $\dfrac{dy}{dx}(0) = 3$.

(8) (10 points) Solve the given initial-value problem. $\dfrac{d^2y}{dx^2} - 9y = 0$, $y(0) = 0$, $\dfrac{dy}{dx}(0) = 3$.

(9) (20 points) This problem is more of a preview of work in this course than a review of earlier work, although the computational aspects should be familiar from your previous work in differential equations. At this point in the course, the problem will seem somewhat unmotivated. The motivation and theoretical framework for the problem will become much clearer later.

(a) Consider the two-point boundary value problem for $y(x)$ given below. The quantity λ appearing in the equation is a positive constant. Are there any values of λ for which the problem does not have a solution?

$$\frac{d^2y}{dx^2} + \lambda y = 0 , \; y(0) = 1 , \; y(1) = 2 .$$

(b) Consider the homogeneous two-point boundary value problem given below, with the same equation but now with homogeneous boundary conditions. As in part (a), λ is a positive constant. Are there any values of λ for which there is a non-trivial solution – i.e., a solution which is not identically zero?

$$\frac{d^2y}{dx^2} + \lambda y = 0 , \; y(0) = 0 , \; y(1) = 0 .$$