Problem 2.41 A surveyor finds that the length of the line $OA$ is 1500 m and the length of line $OB$ is 2000 m.

(a) Determine the components of the position vector from point $A$ to point $B$.
(b) Determine the components of a unit vector that points from point $A$ toward point $B$.

**Solution:** We need to find the coordinates of points $A$ and $B$

\[
\mathbf{r}_{OA} = [1500 \cos 60^\circ \hat{i} + 1500 \sin 60^\circ] \text{ (m)}
\]

\[
\mathbf{r}_{OB} = 750 \hat{i} + 1299 \hat{j} \text{ (m)}
\]

Point $A$ is at $(750, 1299)$ (m)

\[
\mathbf{r}_{OB} = 2000 \cos 30^\circ \hat{i} + 2000 \sin 30^\circ \hat{j} \text{ (m)}
\]

\[
\mathbf{r}_{OB} = 1732 \hat{i} + 1000 \hat{j} \text{ (m)}
\]

Point $B$ is at $(1732, 1000)$ (m)

(a) The vector from $A$ to $B$ is

\[
\mathbf{r}_{AB} = (x_B - x_A) \hat{i} + (y_B - y_A) \hat{j}
\]

\[
\mathbf{r}_{AB} = 982 \hat{i} - 299 \hat{j} \text{ (m)}
\]

(b) The unit vector $\mathbf{e}_{AB}$ is

\[
\mathbf{e}_{AB} = \frac{\mathbf{r}_{AB}}{||\mathbf{r}_{AB}||} = \frac{982 \hat{i} - 299 \hat{j}}{1020.0} \approx 0.957 \hat{i} - 0.291 \hat{j}
\]

Problem 2.50 Four forces act on a beam. The vector sum of the forces is zero. The magnitudes $|F_A| = 10$ kN and $|F_C| = 5$ kN. Determine the magnitudes of $F_A$ and $F_D$.

**Solution:** Use the angles and magnitudes to determine the vectors, and then solve for the unknowns. The vectors are:

\[
F_A = |F_A| \hat{i} \cos 30^\circ + j \sin 30^\circ = 0.866|F_A| \hat{i} + 0.5|F_A| \hat{j}
\]

\[
F_B = 6 - 10j, \quad F_C = 0 + 5j, \quad F_D = -|F_D| \hat{i} + 0j.
\]

Take the sum of each component in the $x$- and $y$-directions:

\[
\sum F_x = (0.866|F_A| - |F_D|) \hat{i} = 0
\]

and

\[
\sum F_y = (0.5|F_A| - (10 - 5)) \hat{j} = 0.
\]

From the second equation we get $|F_A| = 10$ kN. Using this value in the first equation, we get $|F_D| = 8.7$ kN.
Problem 2.54  The cables A, B, and C help support a pillar that forms part of the supports of a structure. The magnitudes of the forces exerted by the cables are equal: \( |F_A| = |F_B| = |F_C| \). The magnitude of the vector sum of the three forces is 200 kN. What is \( |F_A| \)?

**Solution:** Use the angles and magnitudes to determine the vector components, take the sum, and solve for the unknown. The angles between the cables and the pillar are:

\[
\theta_A = \tan^{-1}\left(\frac{4 \text{ m}}{6 \text{ m}}\right) = 33.7^\circ.
\]
\[
\theta_B = \tan^{-1}\left(\frac{5 \text{ m}}{8 \text{ m}}\right) = 35.1^\circ.
\]
\[
\theta_C = \tan^{-1}\left(\frac{12 \text{ m}}{8 \text{ m}}\right) = 63.7^\circ.
\]

Measure the angles counterclockwise from the \( x \)-axis. The force vectors acting along the cables are:

\[
F_A = |F_A| (\cos 33.7^\circ + j \cdot \sin 33.7^\circ) = 65.84|F_A| j - 81.14|F_A| j
\]
\[
F_B = |F_B| (\cos 35.1^\circ + j \cdot \sin 35.1^\circ) = 67.97|F_B| j - 90.00|F_B| j
\]
\[
F_C = |F_C| (\cos 63.7^\circ + j \cdot \sin 63.7^\circ) = 93.94|F_C| j - 144.72|F_C| j
\]

The sum of the forces are, noting that each is equal in magnitude,

\[
\sum F = (2.348|F_A| j - 1.8795|F_A| j)
\]

The magnitude of the sum is given by the problem:

\[
200 = |F_A| \sqrt{(2.2489)^2 + (1.8795)^2} = 2.63|F_A|
\]

from which \( |F_A| = 68.24 \text{ kN} \).

Problem 2.117  The rope \( AB \) exerts a 50.0 N force \( T \) on collar \( A \). Determine the vector component of \( T \) parallel to the bar \( CD \).

**Solution:** We have the following vectors

\[
ev_{AB} = (0.2i - 0.3j + 0.25k) \text{ m}
\]
\[
ev_{AD} = \frac{vev_{CD}}{|vev_{CD}|} = (-0.45i - 0.68j + 0.57k) \text{ m}
\]
\[
ev_{CD} = (0.5i + 0.1j) \text{ m}
\]
\[
ev_{AD} = \frac{vev_{CD}}{|vev_{CD}|} = (0.44i + 0.3j) \text{ m}
\]
\[
ev_{CD} = (0.2i + 0.2j + 0.3k) \text{ m}
\]
\[
ev_{AB} = \frac{vev_{CD}}{|vev_{CD}|} = (-0.39i + 0.37j + 0.06k) \text{ m}
\]
\[
ev_{AD} = \frac{vev_{CD}}{|vev_{CD}|} = (0.67i + 0.73j + 0.07k) \text{ m}
\]

We can now write the force \( T \) and determine the vector component parallel to \( CD \).

\[
T = 50 \text{ N}
\]
\[
e_{AB} = -0.35i + 0.37j + 0.06k
\]
\[
e_{CD} = (0.44i + 0.3j)
\]
\[
e_{AB} + e_{CD} = (3.43i + 5.1j - 4.29k) \text{ N}
\]

Problem 2.118  In Problem 2.117, determine the vector component of \( T \) normal to the bar \( CD \).

**Solution:** From Problem 2.117 we have

\[
T = (-33.7i + 36.7j + 2.03k) \text{ N}
\]
\[
T_P = (3.43i + 5.1j - 4.29k) \text{ N}
\]

The normal component to then

\[
e_{AB} = T - T_P = (-37.1i + 31.8j + 3.22k) \text{ N}
\]