Numerical Simulation of the Mechanical Behavior of *Opus Caementicium*

Opportunities and Challenges

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Outline

- Objectives of Research Program
- Simple Collapse Mechanism of Vaulted Structures
- Case Study: the Grande Aula of the Markets of Trajan
- Mechanics of Modern Concrete
- Mechanics of Opus Caementicium
- Modeling of Opus Caementicium
- Conclusions and Future Directions
Research Objective:
The Engineering of Vaulting Systems in Opus Caementicium

Analyze structural behavior to understand how functionality and efficiency are achieved while avoiding structural failure.

Explore how structural thinking developed and design philosophy evolved from analysis of structural damage and failure.

Investigate how the construction process influenced the design solution so as to avoid structural failure during construction.
Analysis of structural behavior

Stress analysis determines the local distribution of tensile and compressive stresses within the structure generated by gravitational loads, support displacements, environmental factors, seismic loads, etc.

With appropriate material modeling, it can also determine the conditions for crack initiation, propagation, and arrest.
Opus Caementicium - Cross Vaulting Carried by Piers Collapse Mechanism

Elastic deformation of concrete vault under gravitational load

Infinitesimal deformations (mm over 10m span)
Opus Caementicium - Cross Vaulting Carried by Piers
Collapse Mechanism

- Elastic deformation of concrete vault under gravitational load
- Infinitesimal deformations (mm over 10m span)
- Bending deformation at the crown of the vault
- Extrados in compression
- Tension at the intrados

Compression
Tension
Opus Caementicium - Cross Vaulting Carried by Piers Collapse Mechanism

Relative motion of the supporting blocks (travertine) increases downward deformations and tensions at the crown
Opus Caementicium - Cross Vaulting Carried by Piers
Collapse Mechanism

- Concrete block
- Fracture plane
- Highest tension

Tensions (or compressions) higher than concrete ultimate stress will fracture the material.

Fracture due to tensile stress propagates on plane orthogonal to the direction of highest tension.

Presence and orientation of large aggregate affects crack propagation (direction and rate of growth).
Opus Caementicium - Fracture Induced Collapse Mechanism

Fractures weaken the structure by reducing the load paths, changing the static and dynamic behavior.

Functionality may be compromised.

Fractures may lead to catastrophic collapse.

Since macroscopic fractures are detectable to the naked eye, they may provide feedback to the structural designer.

The mechanics of fracture in opus caementicium is unknown.

Hadrian’s Villa - Small Baths
Case study: Markets of Trajans - Grande Aula

Static analysis under gravitational loads and motion of the supports

Opus caementicium modeled as linear elastic material

Linear and nonlinear finite element analysis
Grande Aula - Structural System

- Structural skeleton (contrasting walls)
- Concrete vault
- Supporting blocks (travertine)
- Contrasting arches
Grande Aula - Finite Element Models
Grande Aula - Fracture of the Vault

Tensile stresses at the intrados of the vault crown

High tensile stresses may cause a longitudinal crack at the intrados of the crown
Grande Aula - Fracture of the Vault

Excellent correlations between tensile stresses (model) and repaired cracks (reality)

Similar fracture patterns on the lateral vaults
Grande Aula - Contrasting Arches

Presence of the arch does not affect stresses (tensions) at the crown of the vault (under static loading)

Removing the arch has only a local effect on stress distribution

Arch does not provide contrasting function
Grande Aula - Contrasting Arches

Computed stress fields show high tensile stresses at the attachments of the arch.

Fractures at the arch springing and evidence of major reconstruction are visible on the actual arches.
Grande Aula - Research Hypotheses

The Grande Aula is a limit case of cross-vaulting design in *opus caementicium* derived from previous building practices (with different materials?)

This design is not adequate (produces structural failures) for larger scale cross-vaults (such as the Frigidarium of Diocletian’s Baths)

The analysis and the correction of the structural deficiencies revealed by the limit case (Grande Aula) provide the basis for a new (and mature) cross-vaulting design (Frigidarium)

At the present time, these hypotheses cannot be tested in full through structural numerical modeling due to the limitations in representing the mechanical behavior of *opus caementicium*
Mechanical Behavior of Modern Concrete *

Composite material consisting of different size aggregate particles embedded in a cement paste matrix (mortar)

Excellent mechanical strength in compression but comparatively poor in tension

Aggregate-mortar interface constitutes the weakest link in the composite system and the primary cause of for the low tensile strength

A study of the mechanics of modern concrete serves as a valuable point of departure for understanding the behavior of opus caementicium

Mechanical Testing of Concrete - Compression

\[ \text{stress} = \frac{P}{A} \]

\[ \text{strain} = \frac{\Delta L}{L} \]

Slow loading and unloading below ultimate strength (max stress)
Response: nonlinear and only partially elastic (elastic modulus?)
Mechanical Behavior of Concrete - Compression

Cement paste (mortar) and aggregate - tested separately - exhibit a sensibly linear stress-strain behavior.

Aggregate is considerably stiffer than cement paste.

Before loading, concrete contains a large number of microcracks, due to segregation, shrinkage, or thermal expansion in the cement paste.

During loading, new cracks develop especially at the interfaces cement-aggregate due to differential stiffness.

Propagation of microcracks during loading is the primary cause of nonlinear stress-strain behavior of concrete and the loss of energy (hysteresis) during unloading.
Mechanical Behavior of Concrete - Compression

Accumulation of microcracks increases local stress concentrations and reduces the number of load carrying paths.

Peak of stress-strain curve defines ultimate strength of the specimen.

After peak, nominal stress decreases while strain continue to increase (pseudo-ductile behavior).

At about 75% ultimate strength, fractures propagate in an unstable manner.

Macroscopic cracks appear during the descending part of stress-strain curve.

Unloading-reloading curves show widening of hysteresis and degradation of stiffness.
Mechanical Behavior of Concrete - Tension

Stress-strain curve similar to compression.

Tensile stresses induce early crack propagation - in direction transverse to stress direction.

Ultimate strength in tension approx 5% to 10% of ultimate strength in compression.

Aggregate-mortar interface constitutes the weakest link in the composite system and the primary cause of low tensile strength.
Mechanical Testing of Opus Caementicium*- Compression

Specimens taken from Trajan’s port, Hadrian’s villa, and Basilica of Maxentius

Stress-strain curve similar to modern concrete (nonlinear response and hysteresis likely induced by fracture propagation)

Ultimate strength in compression: avg = 4.55 MPa, range = 1 - 6.7 MPa, generally correlated with quality of concrete confection, best quality found in highly loaded areas

Mechanical Testing of Opus Caementicium*- Compression

Wall specimens taken from Köln, Saalburg, Trier

No stress-strain curve provided

Wide variation of ultimate strength in compression (avg = 12.9 MPa, range = 6.1 - 26 MPa)

Strength considerably higher than S F values (possibly due to diversity in mortar, aggregate size, specimen size)

Mechanical Testing of Mortar*- Compression

Specimens of modern lime/pozzolana/water mortar (ratios 1/3/1.4 by volume) cured in water tested in compression and tension after 7, 28, 90, 180, 360 days. Results for each series show very small spread (7% max).

Most of curing reached at 180 days - nearly linear elastic behavior, little ductility.

Compressive ultimate strength increases substantially with age (to more than 100% higher than opus caementicum).

Mortar exhibits high ductility (plasticity) up to 28 days (suggests adaptability of opus caementicum to deformations of formwork and supports during construction).

**Mechanical Testing - Tension, Elastic Modulus, and Density**

### Opus Caementicium*

- **Tensile ultimate strength in *opus caementicium*** measured only in two specimens both from Hadrian’s Villa: 0.7 and 0.9 MPa (approx. 10% of compressive strength)

- **Wide variation of elastic modulus in compression**: avg = 3530 MPa, range = 800 - 9170 MPa, highest values correspond to best quality of concrete confection

- **Density**: avg = 15.55 kN/m³, range = 13.5 - 17.7 kN/m³, highest values correspond to best quality of concrete confection

### Mortar*

- **Tensile ultimate strength**: 7 days = 0.84 MPa, 90 days = 1.33 MPa, 360 days = 0.95 MPa

- **Elastic modulus in compression**: 7 days = 938 MPa, 90 days = 2960 MPa, 360 days = 3350 MPa

### Tuff (*tufo lionato)**

- **Compressive ultimate strength**: 28.5 MPa (dry), 15.9 MPa (water saturated)

- **Elastic modulus**: 10,000 MPa (dry), 6,000 MPa (water saturated)

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Opus Caementicum - Theoretical Constitutive Model

Model assumes identical linear elastic behavior in compression and tension.

- Elastic modulus = 3500 MPa
- Ultimate strength in compression = 4.5 MPa
- Ultimate strength in tension = 0.45 MPa
- Mass density = 1500 kg/m³

Compatible with experimental data up to ultimate strength.

- Does not include energy loss during reloading.
- Does not include post-ultimate (plastic) behavior.
- Does not include time-dependent behavior.
Opus Caementicium - Theoretical Constitutive Model

Model assumes identical linear elastic behavior in compression and tension

Appropriate for STATIC ANALYSIS:
-- can predict the onset of critical stress state
-- but cannot follow the evolution of critical state to collapse

Limited applicability for DYNAMIC ANALYSIS:
-- applicable to modal extraction (natural frequencies)
-- not applicable to advanced earthquake analysis

Not applicable for VISCOPLASTIC ANALYSIS
-- cannot model mortar before curing
-- cannot analyze stress state during construction
Conclusions and Future Directions

At Rochester we are developing a theoretical and computational model to represent the fracture and its propagation in structures built in opus caementicum.

A comprehensive experimental study is necessary to provide the parameters for the model and to validate its implementation.

This model will allow us to expand the scope of our investigation in Roman design practices for monumental vaulting systems.

For particular structures, the new model will allow us to explore different mechanisms of collapse under static (gravitational) and dynamic (seismic) loads.

This model could be useful for addressing problems of conservation in existing monuments.